

Journal of Financial and Actuarial Mathematics and Management

A joint edition of the Institute of Information Sciences and Institute of Social Sciences of Perun Academy
Publishing Company: PSP Publishing
ISSN: 2367-5810
Issue 6, Year VI (2018)
Founding Editor and Editor-in-Chief: Pavel Stoynov

Editorial Board: Hiroyuki Ono (Japan), Yasukazu Yoshizawa (Japan), Naoyuki Ishimura (Japan), Boris Melnikov (Russia), Elena Melnikova (Russia), Goran Karanovic (Croatia), James Osondu (Nigeria)

Contents

P. Stoynov. The Chinese dynastic cycle – historical and quantitative overview……..…….3
A. Abdollahi, F. Riahi, S. Saadi. The effect of client identity on audit quality in Iran…….……...8
D. Gotseva, R. Trifonov, P. Stoynov. Artificial intelligence and advanced technologies - some case studies from European research space……..……..……15
P. Todorov. Additive processes with return to zero and Stoynov processes……..…….……...…….....……..…22

The Chinese dynastic cycle – historical and quantitative overview

Pavel Stoynov
Sofia University "St. Kliment Ohridski", Sofia, Bulgaria,
Perun R&D Academy, Sofia, Bulgaria

Abstract.
The traditional Chinese historiography divides the history of China into dynastic periods and explains this division by the dynastic cycle theory. According to this theory, the dynasties are founded by strong leaders but the next generations of emperors can't keep the high level of the ruler competence needed and, because of this, the dynasties gradually decay and die.

This paper considers the durations of the most typical Chinese imperial dynasties. In the context of the dynastic cycle theory, a brief historical overview of the Chinese dynasties is performed and some statistical measures and distribution fits are applied to obtain general conclusions about the Chinese dynastic cycle.

Key words: dynastic cycle, China, Chinese dynasties

Introduction

The traditional Chinese historiography divides the history of China into dynastic periods and explains this division by the dynastic cycle theory. According to this theory, the dynasties are founded by strong leaders but the next generations of emperors can't keep the high level of the ruler competence needed and, because of this, the dynasties gradually decay and die.

This paper considers the durations of the most typical Chinese imperial dynasties. In the context of the dynastic cycle theory, a brief historical overview of the Chinese dynasties is performed and some statistical measures and distribution fits are applied to obtain general conclusions about the Chinese dynastic cycle.

Dynastic cycle is an important political theory in Chinese history. According to this theory, each dynasty rises to a political, cultural, and economic peak and then, because of moral corruption, declines, loses the Mandate of Heaven, and falls, only to be replaced by a new dynasty. The cycle then repeats under a surface pattern of repetitive motifs (Dillon, 1998).

It sees a continuity in Chinese history from early times to the present by looking at the succession of empires or dynasties, implying that there is little basic development or change in social or economic structures (Reischauer, 1965). Some historians expressed their doubts that "the concept of the dynastic cycle... has been a major block to the understanding of the fundamental dynamics of Chinese history" (Fairbank and Reischauer, 1960).

The cycle appears as follows:

1. A new ruler unites China, founds a new dynasty, and gains the Mandate of Heaven.
2. China, under the new dynasty, achieves prosperity.
3. The population increases.
4. Corruption becomes rampant in the imperial court, and the empire begins to enter decline and instability.
5. A natural disaster wipes out farm land. The disaster normally would not have been a problem, but together with the corruption and overpopulation, it causes famine.
6. The famine causes the population to rebel and a civil war ensues.
7. The ruler loses the Mandate of Heaven.
8. The population decreases because of the violence.
9. China goes through a warring states period.
10. One state emerges victorious.
11. The state starts a new empire.
12. The empire gains the Mandate of Heaven.

The Mandate of Heaven was the idea that the Emperor was favored by Heaven to rule over China. The Mandate of Heaven explanation was championed by the Chinese philosopher Mencius during the Warring States period. It has 3 main phases:

1. The beginning of the dynasty.
2. The middle of the dynasty's life and the peak of the dynasty.
3. The decline of the dynasty, both politically and economically, until it finally collapses (Reischauer, 1965).

The Chinese dynasties – short historical overview

The main dynasties in the Chinese history are presented in Table 1. It may be seen that some dynasties have overlapping periods, due to the fact that they existed in parallel. In other cases, there are gaps between two consecutive dynasties in the list due to the fact that there were periods of decentralization.

Here we present brief historical overview of the dynasties.

Xia dynasty is the most powerful among several small states in the valley of Huang He. Its founder is the legendary Yu. It exists 550 years, between 1900 and 1350 BC.

Shang dynasty existed in parallel with Xia. After, Shang conquered Xia. From 1766 till 1122 BC, Shang period lasts 644 years – the longest dynastic period in Chinese history.

Chou dynasty coexisted with Shang for a long period. It was located to the west of Shang. After Chou conquered Shang, the dynasty extended its territory in North China. Chou is divided into two periods – Western (Early) Chou and Eastern (Late) Chou. Eastern Chou receives its name because the dynasty moved its capital to east, to Luoyang as the Jouanjouan tribe invaded Chou. Western Chou lasted 351 years. Eastern Chou is divided into two periods Spring and Autumn (771-481 BC) and Warring Kingdoms (481-221 BC). This dynasty existed 195 years.

The Qin dynasty lasted only 15 years but it initiated a series of campaigns leading to the end of Warring Kingdoms period and unification of China.

The Han dynasty is divided into Early (Western) and Late (Eastern) Han. Between them for a short time the dynasty Sin came to power which is the shortest dynasty in our list. The capital of Early Han is Changan, about 480 km to the west of Luoyang, the capital of Late Han, from where the name.

The Sui dynasty is created in 589 year by Wan (589-604). The successor of Wan, his second son Yang (604-618) starts catastrophic wars against Korea, and finally is killed by the creator of Tang – Li Yuan.

Tang dynasty (626-907) is one of the longest Chinese dynasties. It lasted 281 years.

Sung Dynasty includes North and South Sung. North Sung (960-1115) is created by Taizu. It may be said that North Sung existed till 1115 when the dynasty Jin was announced, but also we may consider that North Sung formally existed till 1141 when a peace contract was signed between Jin and Sung according to which Jin rules North China and Sung, called already South Sung, rules South China.

The dynasty Jin (1115-1234) was created by the Jurchen tribes of Manchu-Tungus group. It was defeated by Mongols. Under the pressure of Genghis khan, Jurchens move their capital from Yanjing (Beijing) to Kaifeng. After, Ogedei khan conquered Kaifeng. The last Jin emperor tried to obtain help from South Sun, but he did not receive it and killed himself.

South Sun (1127-1279) is destroyed by Kublai khan who created in 1260 year the Mongol dynasty Yuan (1260 -1368).

Yuan was succeeded by Ming (1368 -1634) which in its tern was conquered by the Manchurian dynasty Qing (1634-1911) – the last dynasty of the imperial China.

Table 1. The main Chinese dynasties – modeling with ST distributions.

Dynasty name	Period	Length of the period	D_i^n for ST1(46,228)	D_i^n for ST1(2,28)
Xia	1900-1350 BC	550	39	1
Shang	1766-1122 BC	644	46	1
Western Chou	1122-771 BC	351	25	2
Eastern Chou	771-221 BC	290	21	1
Qin	221-206 BC	15	1	2
Western Han	206 BC – 9	215	15	1
Sin	9-23	14	1	1
Eastern Han	25-220	195	14	2
Sui	589-618	29	2	1
Tang	626-907	281	20	1
North Sung	960- 1115	155	11	1
Jin	1115-1234	119	9	1

South Sung	1127-1279	152	11	1
Yuan	1260 -1368	108	8	1
Ming	1368 -1634	266	19	2
Qing	1634-1911	277	20	1

The Chinese dynasties – statistics about the dynastic cycle

The data in Table 1 shows that the shortest dynasty is Sin dynasty with a period of 14 years. The longest dynasty is Shang dynasty with a period of 644 years. The average duration of a dynasty is 228 years. The standard deviation is 177 years.

If we consider separately Chinese dynasties in the period BC and Chinese dynasties in the period AD, calling the dynasties of the first period Old dynasties and the dynasties of the second period New dynasties, and considering Western Han as belonging of both dynastic types, then we may consider the basic statistics for these two types of dynastic cycles.

The Old dynasties have shortest and largest periods again equal to 15 years and 644 years correspondingly. The average dynastic period is 344 years. The standard deviation is 228 years.

The New dynasties have shortest period 14 years (Sin dynasty) and longest period 281 years (Tang dynasty). The average dynastic period is 164 years and the standard deviation is 93 years.

If we consider the dynastic period as a random variable, it is natural to try to find reasonable probabilistic distribution of this variable.

One possibility is to consider the dynastic cycle as exponentially distributed random variable, i.e. random variable $\xi \in Exp(\lambda)$ with probability density function

$$f_\xi(x) = \begin{cases} \frac{1}{\lambda} e^{-\frac{x}{\lambda}}, \ x \geq 0 \\ 0, \ x < 0. \end{cases}$$

In this case $E\xi = \lambda = 228$ for the whole Chinese history and the standard deviation is $\sqrt{D\xi} = \lambda = 228$ from one side (if we suppose exponential distribution) and $\sqrt{D\xi} = \lambda = 177$ (as estimated) from other. Generally, as 228 and 177 are relatively close (but with small precision), we may accept the hypothesis that dynastic cycle is exponentially distributed, i. e. $\xi \in Exp(228)$.

A more realistic hypothesis could be that the dynastic cycle has gamma distribution, i. e. the distribution of random variable $\xi \in \Gamma(\alpha, \beta)$ with probability density function

$$f_\xi(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}, \ x \geq 0 \\ 0, \ x < 0 \end{cases}.$$

Here $\Gamma(\alpha)$ is defined by $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx.$ In the case of gamma distribution, the mean is $E\xi = \alpha\beta = 228$ and the standard deviation is $\sqrt{D\xi} = \sqrt{\alpha\beta^2} = 177.$ By solving the system of equations

$$\begin{cases} \alpha\beta = 228 \\ \sqrt{\alpha\beta^2} = 177, \end{cases}$$

we obtain $\alpha = \left(\frac{228}{177}\right)^2 = 1{,}66$ and $\beta = \frac{228}{\alpha} = 137.$ So, the dynastic cycle is approximated by a random variable $\xi \in \Gamma(1{,}66; 137).$

Another suitable distribution for modeling dynastic cycles is so called ST distribution (Stoynov, 2013) . We say that a random variable ξ with probability mass function $f_\xi(x)$ has distribution of $ST(n, \beta)$ family and denote this fact $\xi \in ST(n, \beta),$ if the probability mass function of ξ is given by the formula

$$f_\xi(x)=\begin{cases}\sum_{k=1}^{n+1}P(D^n=k)\,f_\xi(x\mid D^n=k)=\sum_{k=1}^{n+1}P(D^n=k)\,f_{G^k}(x), x\geq 0\\ 0, x<0.\end{cases}$$

where G^k are random variables with probability mass function $f_{G^k}(x)=f(k,\beta)$ and D^n are positive integer mixing random variables.

Here for G^k we may adopt different families of distribution.

In this article, the case of ST family of first kind is considered where

$$G^k\in\Gamma(k,\frac{1}{\beta})\equiv Erlang(k,\frac{1}{\beta}),$$

i. e. $\xi\mid D^n\equiv Erlang(D^n,\frac{1}{\beta}).$ Correspondingly, D^n is a random variable, taking values $k=1,\ldots,(n+1)$ with probabilities $P(D^n=k)=\frac{C(n,\beta)n!}{\beta^k(n-k+1)!},k=1,\ldots,(n+1)$ where the coefficients $C(n,\beta)$ are given by the formulas:

$$C(n,\beta)=\frac{1}{I(n,\beta)},$$

$$I(0,\beta)=\frac{1}{\beta},$$

$$I(n,\beta)=\frac{1}{\beta}+\frac{n}{\beta}I(n-1,\beta),n=1,2,\ldots$$

If a random variable has ST distribution of first kind, we will denote this fact $\xi\in ST1(n,\beta).$ For such a variable, the probability density is density of Erlang distribution with first parameter being another integer-valued random variable with a specific distribution..

If in $ST1(n,\beta)$ we enforce $D^n=k,$ i. e.

$$P(D^n=k)=1,P(D^n=i)=0,1\leq i\leq k-1<k+1\leq i\leq n+1,$$

which may be considered as degenerate ST distribution of first kind, we actually obtain Erlang distribution, i. e. $\xi\in Erlang(k,\frac{1}{\beta}).$

The exponential distribution is a special kind of $ST1(n,\beta)$ distribution are $ST1(0,\beta)\equiv Exp(\beta)$.

The case $n=2$ leads to $ST1(2,\beta)$ distribution which can be defined as weighted exponential distribution by the weight function $w(x)=(1+x)^2.$

We say that the random variable ξ has $ST1(2,\beta)$ distribution and denote $\xi\in ST1(2,\beta)$, if its probability mass function is given by

$$f_\xi(x)=\begin{cases}\frac{\beta^3}{\beta^2+2\beta+2}e^{-\beta x}(1+x)^2, x\geq 0\\ 0, x<0.\end{cases}$$

Let us suppose that there are two types of events related to dynasty change. The first type is “condition for change of the dynasty”. Intervals between two conditions for change are exponentially distributed. The second kind of events we call “real change of dynasty”. The interval between two real changes of the dynasty – the dynastic cycle – can be modeled by $ST1(n=[\frac{t_{\max}}{t_{\min}}],\beta)$ distribution where $t_{\max}$ is the maximal and $t_{\min}$ - the minimal observed dynastic period and $[.]$ is the integer part of a number.

For Chinese dynasties we may estimate that $\frac{1}{\beta} = 228.$ Also, $n = [\frac{t_{\max}}{t_{\min}}] = \frac{644}{14} = 46$ and the realizations of the random variables D_i^n are as shown in Table 1, forth column.

Another approach for estimating the distribution of the dynastic cycle with ST distribution is based on the observation that some of the dynasties have two subperiods. For example, Chou and Han have Eastern and Western periods and Sung has North and South periods. Such sub-periods appear when some risk event for the dynasty happened and the dynasty changed its characteristics – moved to another territory, changed rulers and/or ruling ethnos, changed its capital etc. This usually happens when the dynasty survives after big danger and obtains the chance for a second cycle.

In this case, we estimate the distribution of the dynastic cycle with $ST1(2, \beta)$ distribution. We suppose that there are two types of events: "change to new dynasty" and "change of dynasty or sub-period of a dynasty". The interval between changes of a dynasty or sub-period of a dynasty is exponentially distributed. The interval between changes to new dynasties is with $ST1(2, \beta)$ distribution

Then we have $\frac{1}{\beta} = 228$ and also $n = 2$ and the realizations of the random variables D_i^n are as shown in Table 2, fifth column.

Natural disasters (mainly floods of Huang He river) are included in the theory of the dynastic cycle. The ST-distribution can also be used to model inter-arrival times between two consecutive floods of this river (Stoynov, Zlateva, Velev, 2015).

Conclusion

There are different ways for estimating the distribution of the dynastic cycle. One flexible way is to use ST distribution. Further development could be to compare in formal way different ways of distribution fitting. Also, more complete list of dynasties may be used or list of dynasties can be replaced by list of rulers.

References

Dillon, M. (1998). China: A Historical and Cultural Dictionary. Richmond, Surrey: Curzon. ISBN 978-0700704392., p. 87

Fairbank, J. and E. Reischauer (1960) East Asia: The Great Tradition. Boston, p. 115.

Reischauer, E. (1965). The Dynastic Cycle, in John Meskill, The Pattern of Chinese History, Boston: D.C. Heath and Company, pp. 31-33.

Stoynov, P. (2013) Stoynov Process – Definition and Simulations. – Proceedings of the Sixth International Conference Financial and Actuarial Mathematics, Sofia, Bulgaria, 97-104.

Stoynov, P., Zlateva, P, Velev, D. (2015) An Approach for Modelling Inter-arrival Time of Floods. – International Journal of Innovation, Management and Technology, vol. 6, no. 4, 267-271. http://www.ijimt.org/vol6/613-S014.pdf

The effect of client identity on audit quality in Iran

Ahmad Abdollahi
Department of Accounting, Golestan Institute of Higher Education,
Gorgan, Iran
Email: ahmabdollahi@gmail.com
Fatemeh Riahi
Department of Accounting, Golestan Institute of Higher Education,
Gorgan, Iran
Email: fateme.riahi@yahoo.com
Soraya Saadi
Department of Accounting, Golestan Institute of Higher Education,
Gorgan, Iran
Email: saadisoraya@yahoo.com

Abstract.
According to Social Identity Theory (SIT), people tend to classify themselves and others into various social categories, such as organizational membership, religious affiliation, gender, and age cohort. As these examples suggest, people may be classified in various categories, and different individuals may utilize different categorization schemas. Categories are defined by prototypical characteristics abstracted from the members. Although client familiarity is desirable from the auditor's perspective, identifying with clients and client identity threatens auditor objectivity. The aim of this paper is explanation about the extent to which auditors identify with clients, the effect of auditor-client identification on auditors' client acquiescence to client-preferred treatment, and, finally, whether the harmful effects of Auditor-client identification can be extended to a broader set of reduced audit quality acts. The results of literature review show that client identity influence on audit quality and it may be serious for auditing profession.
Key words: Social Identity Theory, client identity, audit quality

Introduction

Investors perceive a lower quality of accounting information when the audited company's management exerts influence on auditors (Dhaliwal, Gleason, Heitzman, & Melendrez, 2008; Lambert, Leuz, & Verrecchia, 2007). At the same time, valueadded audit services e i.e., clienteservice activities resulting from an audit that are not directly related to verifying the financial statements, such as providing information and advice beyond core audit services e are found to be most abundant when the auditor has a strong relationship with the client (Herda & Lavelle, 2013). In addition, clients prefer a relational approach from their auditors (Fontaine & Pilote, 2011, 2012), but client desire to have a committed and relationally oriented auditor appears difficult to reconcile with the need for auditor objectivity (€Ohman, H€ackner, & S€orbom, 2012). The present study addresses this problematic issue by examining the impact of auditoreclient identification on auditor objectivity and audit quality by applying social identity theory to the auditoreclient relationship. Bamber and Iyer (2002, 2007) suggested that auditors identify with their clients and that this identification reduces auditor objectivity in such a way that an identifying auditor tends to acquiescence to the client's preferred accounting position.

The Independence Standards Board identifies auditors' familiarity with the client as one of five threats to auditor independence (ISB 2000a).To foster auditor independence and objectivity, the Sarbanes-Oxley Act of 2002 bans auditors from various consulting activities, tightens partner rotation requirements, and raises the issue of accounting firm rotation. The untested assumption behind these new regulations is that the close ties between auditor and client are inappropriate because they impair auditors' objectivity in performing the audit, which in turn contributes to perceived audit failures such as Waste Management, Global Crossing, MicroStrategy, and Enron. Yet auditors must be familiar with their client and its management to understand the client well enough to plan and perform an effective and efficient audit (Auditing Standards AU 311). This conflict between: (1) auditors'need to be familiar with the client in order to perform the audit, versus (2) the threat to objectivity from this familiarity, have led critics to argue that it is not possible to expect auditors to exercise objective, unbiased judgment (e.g., Bazerman et al. 2002).

We extend this research by examining a broader set of behaviours. In particular, we examine whether the effects of auditoreclient identification extend to reduced audit quality (RAQ) acts. The RAQ acts considered here constitute unethical time-saving acts having no immediate relationship to client preferences, and previous research has found that RAQ acts arecaused primarily by time budget pressure (TBP) (e.g., Coram, Ng, &Woodliff, 2003; Otley & Pierce, 1996a; Pierce & Sweeney, 2004;Willett & Page,1996). However, Cianci and Bierstaker (2009) argued that the underlying mechanism of the relationship between TBP and RAQ acts is

cognitive bias caused by TBP. Affected by this bias, auditors believe that they can take such timesaving shortcuts as superficially reviewing client documents or false signoff without increasing audit risk. Auditors who identify with their clients are likely to suffer from a similar biasing of their judgment. Consequently, auditoreclient identification is a likely cause of RAQ acts.

The remainder of this paper is made up of two parts and then is ends with statements ; firstly,it introduces two essential charts for understanding social identity theory and justifying costomer identity , then the third section examines the professional recognition reform framework for the acceptance of audit clients and the reduction of audit quaiity and concludes the last part of this article.

Social Identity Theory and the justification for the client identity construct

Whenever HQ managers are assigned abroad to work as expatriates, they enter a social milieu, namely the foreign country, in which they have to interact with a series of people, including local staff (HCNs). How will they see themselves in their interactions with local staff and how will local staff view them? This is what sociologists mean by identity (Haslam et al., 1999). Valsiner (2007) defines identity as the sundry ways in which individuals see themselves and others within their social milieu. This can also be described as the different selves that individuals adopt in their social interactions. Identity is constructed on multiple spheres or levels (Cole, 1996; Valsiner, 2007) and there are three basic types of identity. The first is personal identity (Perry, 2008), the aspects of individuals that define their personality. In the case of our two focal groups, personal identitywould apply to the manner inwhich individualmembers behave or interrelatewith others (e.g.,whether they are outgoing, polite, trustworthy, optimistic, etc.). The second is role identity (Ashforth, 2001),which includes professional or organizational identity. As its name suggests, this refers to the part an individual is expected to play within a group (e.g., manager, auditor, salesperson, etc.). All localmembers of staff have a role to playwithin their milieu (i.e., the subsidiary's environment), and that role will complement the parts played by others (e.g., corporate expatriates). Finally, there is social identity, which is imposed by demographic variables, such as gender, age, ethnicity, or nationality. Tajfel (1972, 292) identifies social identity as "the individual's knowledge that he belongs to certain social groups together with some emotional and value significance to him of this group membership". People derive their self-concept (identity) largely fromthe social group to which they belong. Social identity is thus containedwithin the individual (Hogg & Abrams, 1988).

Social Identity Theory holds that individuals' social identity results from a selfcategorization process, through which individuals cognitively group themselves with others. As such, Social Identity Theory potentially provides a cognitive-based explanation, in contrast to aeconomic-dependency explanation, for why clients may have too much influence over their auditors. According to the theory, individuals classify themselves into multiple social groups, such as occupation, organization, division, gender, nationality, ethnicity, and age (Turner 1987; Ashforth and Mael 1989). These multiple identities are distinct, and may be compatible or competing with one another (Wallace 1995; Scott 1997). Researchers find organizational identification in a variety of settings, including work groups (e.g., Wan-Huggins et al. 1998), soldiers (Mael and Ashforth 1995), college alumni (Mael and Ashforth 1992), accounting firm alumni (Iyer et al. 1997), journalists (Russo 1998), cooperative extension agents (Scott 1977),and auditors (Bamber and Iyer 2002).

These self-categorizations act as a point of departure for thinking and relating, and so social identity increases the likelihood that the individual internalizes the group's norms and values. Adoption of a particular identity affects the way individuals interpret information and make decisions (Lembke and Wilson 1998). Individuals tend to identify with groups whose values appeal to the individual (Alvesson 2000). Moreover, these self-categorizations may be viewed as separate and distinct identities so that, for example, identification with one's employing organization does not necessarily preclude identification with one's profession (Lachman and Aranya 1986; Wallace 1995; Bamber and Iyer 2002). Nevertheless, individuals who see themselves primarily as professionals are likely to identify less with their employing firm, since the firm is secondary for their identity (Alvesson 2000). Identification affects outcomes. For example, Deetz (1995) reports that professionals who identify with a client are more likely to underreport the actual hours worked on a project because they did not want the client to pay for work performed somewhat inefficiently. Iyer (1998) finds that accounting firm alumni are more likely to steer business to their former accounting firm. King (2002) reports experimental evidence that a sense of social identity among auditors partially counters the self-serving biases suggested to compromise auditors' objectivity, and Towry (2003) finds that team identity affects an incentive system's effectiveness.

Social Identity Theory predicts that service organization employees whose direct interaction with clients is a major part of their work will begin to identify with their clients. For example, Alvesson (2000, 1109) finds that computer consultants who work at the client's location on a daily basis for many months report that "they sometimes knew the client company better than their own employing company and that they experienced identity and loyalty problems." Auditors may work at the same client on a daily basis for long periods and on a recurrent yearly basis. To perform an effective and efficient audit, auditors must understand the client's business,

accounting and information systems, and know its key personnel (AU 311). Auditors may also view a client as a potential future employer. For all these reasons, auditors are likely to identify with their clients.

Three antecedents of auditoreclient identification were examined by Bamber and Iyer (2007), who found that the effects of auditor tenure, client importance, and client image, respectively, on auditor acquiescence were mediated by auditoreclient identification. Supporting a large body of social psychological research, Dutton et al. (1994) argued that the longer a person remains with an organization, the more salient the organizational membership becomes for self-categorization. Bamber and Iyer (2007) confirmed that the length of time that an auditor has worked on the audit engagement (i.e., auditor tenure) is positively related to auditoreclient identification.

Furthermore, auditors will tend to identify more with clients that they deem important. Such clients are often the largest clients of an audit firm (Chung & Kallapur, 2003), and auditors invest more time and effort in important clients, though small clients may be important for non-Big 4 firms due to the relative size of the audited company. Auditors are sometimes recruited by their clients (Dart & Chandler, 2013), a situation that causes auditors to value clients as potential employers as well. Social identity theory and evidence from Bamber and Iyer (2007) suggest that the positive effect on auditors' self-image provided by association with important clients in combination with the amounts of time that auditors allocate to such clients stimulate audit client identification. Finally, social identity theory suggests that people identify with groups that have an appealing image, increasing individual self-esteemand image (Haslam, 2001; Tajfel & Turner,1985). For these reasons, we expect to confirm Bamber and Iyer's (2007) results and find that auditors identify more with clients having a positive image in the eyes of the general public, auditors, or both.

Professional identification, auditors' client acquiescence, and RAQ acts

If auditors do exhibit significant levels of client identification, it is necessary to determine when this affiliation compromises auditor objectivity. Objectivity requires the auditor to make unbiased audit judgments instead of simply acquiescing to the client's wishes (ISB 2000a). Rule 102, Integrity and Objectivity, of the AICPA *Code of Professional Conduct* requires that a member "shall be free of conflicts of interest, and shall not knowingly misrepresent facts or subordinate his or her judgment to others. Objectivity is at the heart of the auditor's value to society: to provide an unbiased opinion on the fairness of the financial statements (Johnstone et al. 2001). Given their professional training, auditors may be able to control the extent of their identification with the client so that it does not impair their professionalism and objectivity. Alternatively, identification with the client may interfere with auditors' objectivity by inducing judgment bias. In a study of lawyers in a large city, Wallace (1995) finds that professionals can be highly committed to both their organization and their profession, and that identification with one does not necessarily affect their identification with the other. Similarly, Bamber and Iyer (2002) find that auditors exhibit relatively low levels of conflict between identification with their employing firm and identification with their profession, suggesting that auditors can manage competing demands from their firm and profession. If auditors can manage these competing demands, perhaps they can also maintain their objectivity despite identifying with the client.

While prior research has not directly examined how client identification affects auditors' judgments, there is reason to believe that it impairs auditors' objectivity. The Independence Standards Board's *A Conceptual Framework for Auditor Independence* (2000a) lists familiarity: "threats that arise from auditors being influenced by a close relationship with an auditee," as one of five threats to auditor independence.5 Johnstone et al. (2001) identify interpersonal relationships between the auditor and client as an incentive that creates a risk to independence.

Moreover, in his analysis of accounting professionalism, Arthur Wyatt (2004, 50) suggests "keeping the client happy and doing what was necessary to retain the client achieved prominencethat did not exist prior to the advent of the successful consulting arms within the firms…… Healthy skepticism was replaced by concurrence. Accordingly, that auditors' identification with their clients impairs their objectivity so that the likelihood auditors acquiesce to the client-preferred treatment increases with the extent to which they identify with the client.

Several auditor behaviours identified as RAQ acts have been examined in previous research, including accepting weak client explanations (Coram et al., 2003; Pierce & Sweeney, 2004; Willett & Page, 1996), superficially reviewing client documents (Dalton & Kelley, 1997; Kelley & Margheim, 1987; Pierce & Sweeney, 2004), and inadequately investigating accounting principles (McNair, 1991; Otley & Pierce, 1996a). Moreover, research has found that auditors in some cases reject awkward-looking items from a sample (Coram et al., 2003; Willett & Page, 1996), inappropriately rely on the client's internal control (Pierce & Sweeney, 2004), fail to pursue questionable items (McNair, 1991), and sign off prematurely (Alderman &Deitrick, 1982).These RAQ acts are closely related to TBP in audit firms and this pressure is the most commonly observed cause of these behaviours. RAQ acts have been so closely associated with TBP that they may be interpreted simply as ways for auditors to save time in order to attain time budgets, but Cianci and Bierstaker (2009) offered an alternative way

of looking at the phenomenon. They demonstrated that TBP causes auditors to suffer from biased judgment, which in turn causes RAQ acts to become more frequent. This judgment bias, i.e., efficiency-mediated estimation, means that auditors discount negative information, emphasize positive information about the client's accounting as more relevant, and make favourable assessments of the client's internal control as demands for efficiency increase. This in turn means that auditors can perform RAQ acts to save time in the belief that these acts do not involve an audit risk increase.Cianci and Bierstaker (2009) explained auditors' biased judgment with reference to motivated reasoning theory (Kunda,1990, 1999). This theory claims that individuals with directional goals conduct biased searches for and overestimate the importance of evidence supporting their preferred conclusion (Ditto, Scepansky, Munro, Apanovitch, & Lockhart, 1998; Lundgren & Prislin, 1998). Consistent with motivated reasoning theory, accounting research has provided evidence that auditors make judgments in favour of client-preferred alternatives in response to efficiency concerns (Bierstaker, Bedard, & Biggs, 1999; Glover, Prawitt, Wilks, & McDaniel, 2005; Haynes et al., 1998) and other evidence suggests that auditors can have the goals of being efficient and supporting client-preferred positions (DeZoort & Lord, 1997; Johnstone, Bedard, & Biggs,2002). Of particular importance in this respect is the similarity of these effects to the inclination of auditors to take a collective's (group or organization) interest to heart (Hogg, 2003; Sedikides & Brewer, 2001; Turner et al.,1987), which is likely to appear when the auditor identifies with clients. In total, these observations suggest that auditors who identify with their clients are more likely to commit RAQ acts than are other auditors. Similarly, Deetz (1995) found that professionals who identify with a client are more likely to under-report time.

While client identification may pose a threat to auditor objectivity, other features of the audit may offset this threat. One factor is auditors' professional identification. Research on the organizationaleprofessional conflict experienced by auditors has claimed that professional commitment or professional identity is separated from and precedes other commitments for this strongly professional group (Aranya & Ferris, 1984; Aranya, Pollock, & Amernic, 1981; Carrington, Johansson, Johed, & €Ohman, 2013; Shafer, Park, & Liao, 2002; Shafer & Wang, 2010). According to the social identity argument, auditors who identify with their profession tend to internalize the values and norms of the profession, their behaviour being highly governed by these values and norms.If it is correct to hypothesize that the propensity to acquiescence to the client's preferred accounting position increases with increasing client identification, the opposite should hold for professional identification. This reasoning is supported by Jenkins and Lowe (1999), who argue that auditors could be either advocates for their clients, supporting strong client identification, or public watchdogs, supporting strong professional identification. For the same reasons that auditors tend to acquiesce less to the client-preferred treatment when they have a stronger professional identity (Bamber & Iyer, 2007), we expect auditors with a stronger professional identity to be less likely to commit various RAQ acts. Auditors who are less independent should have a higher tendency to neglect the audit rigor compromised by RAQ acts.

Discussion and conclusions

This study extends previous literature on the the effect of client identity on audit quality in Iran.

We found that identification with clients varies substantially between auditors, and that client identification is lower than professional identification on average. Partly contrary to this result, Hogg, van Knippenberg, and Rast III (2012) argued that identification across organizational boundaries is difficult to achieve if there are intergroup identity clashes. We recognize that an auditoreclient relationship could well involve intergroup clashes because the auditor's professional identification is strong and being an auditor entails maintaining a certain distance from the client. In the view of researchers such as Fontaine and Pilote (2011, 2012) and Herda and Lavelle (2013), auditors should establish a relational approach to their clients. However, the present study provides further evidence of the impairment of auditor objectivity that may be caused by relaxing the requirement for an arms-length auditoreclient relationship. We found that

auditors' client identification serves to impair auditor objectivity because auditors who identify relatively more with their clients are more likely to acquiesce to the client-preferred treatment of a material accounting issue. This effect was demonstrated using a measure of the extent that the auditor's judgment is biased.We also found that auditors who are more experienced are less likely to acquiesce to the client's position, supporting previous findings (Bamber & Iyer, 2007).Also, Cianci and Bierstaker (2009) found that the effect of TBP on RAQ acts is explained by cognitive bias. They argued that TBP causes the auditor to look more favourably on the client's internal control, and that this assessment by the auditor in turn causes the auditor to inappropriately believe that foregoing some substantive testing, in order to save time, will have less adverse effects.

We found that increasing client identification increases not only the auditor's tendency to acquiesce to the client-preferred treatment of a material accounting position, but that the auditor is more likely to commit a range of RAQ acts.

References

1. Alderman, C. W., & Deitrick, J. W. (1982). Auditors' perceptions of time budget pressures and premature sign-offs: a replication and extension. Auditing: A Journal of Practice and Theory, 1(2), 54e68.

2. Aranya, N., & Ferris, K. R. (1984). A reexamination of accountants' organizationaleprofessional conflict. The Accounting Review, 59(1), 1e15.

3. Aranya, N., Pollock, J., & Amernic, J. (1981). An examination of professional commitment in public accounting. Accounting, Organizations and Society, 6(4), 271e280.

4. Alvesson, M. 2000. Social identity and the problem of loyalty in knowledge-intensive companies. *The Journal of Management Studies* 37(8): 1101-1124.

5. Ashforth, B. E. and F. Mael. 1989. Social identity theory and the organization. Academy of Management Review 14(1): 20-39

6. Ashforth, B. E. (2001). Role transitions in organizational life: An identity-based perspective. Mahwah, NJ: Erlbaum.

7. Bamber, E. M., & Iyer, V. M. (2002). Big 5 auditors' professional and organizational identification: consistency or conflict? Auditing: A Journal of Practice and Theory, 21(2), 21e38.

8. Bamber, E. M., & Iyer, V. M. (2007). Auditors' identification with their clients and its effect on auditors' objectivity. Auditing: A Journal of Practice and Theory, 26(2), 1e24.

9. Bazerman, M. H., G. F. Loewenstein, and D. A. Moore. 2002. Why god accountants do bad audits. Harvard Business Review 80(11): 96-102.

10. Bierstaker, J. L., Bedard, J. C., & Biggs, S. F. (1999). The role of problem shifts on auditors' decision processes in analytical procedures. Auditing: A Journal of Practice & Theory, 18(1), 18e36

11.Carrington, T., Johansson, T., Johed, G., & €Ohman, P. (2013). An empirical test of the hierarchical construct of professionalism and managerialism in the accounting profession. Behavioral Research in Accounting, 25(2), 1e20.

12. Chung, H., & Kallapur, S. (2003). Client importance, non-audit services, and abnormal accruals. The Accounting Review, 78(4), 931e956.

13. Cianci, A. M., & Bierstaker, J. (2009). Auditors' efficiency motivated evaluation. Advances in Accounting, Incorporating Advances in International Accounting, 25, 20e27.

14. Coram, P., Ng, J., & Woodliff, D. R. (2003). A survey of time budget pressure and reduced audit quality among Australian auditors. Australian Accounting Review, 13(1), 38e45.

15. Cole, M. (1996). Cultural psychology: A once and future discipline. Cambridge, MA: Harvard University Press.

16. Dalton, T., & Kelley, T. (1997). A comparison of dysfunctional behaviours by tax accountants and auditors under time budget pressure. Advances in Accounting, 15, 193e208.

17. Dart, E., & Chandler, E. (2013). Client employment of previous auditors: shareholders' views on auditor independence. Accounting and Business Research, 43(3), 205e224.

18. Deetz, S. (1995). Transforming communication, transforming business: Building responsive and responsible workplaces. Cresskill, NJ: Hampton Press.

19. DeZoort, F. T., & Lord, A. T. (1997). A review and synthesis of pressure effects research in accounting. Journal of Accounting Literature, 16, 28e85.

20. Dhaliwal, D. S., Gleason, C. A., Heitzman, S., & Melendrez, D. D. (2008). Auditor fees and cost of debt. Journal of Accounting, Auditing & Finance, 23(1), 1e22.

21. Ditto, P. H., Scepansky, J. A., Munro, G. D., Apanovitch, A. M., & Lockhart, L. K. (1998). Motivated sensitivity to preference-inconsistent information.

22. Fontaine, R., & Pilote, C. (2011). An empirical study of Canadian companies to determine clients' preferred relationship approach with their financial auditor. Journal of Marketing Trends, 1(7), 61e69.

23. Fontaine, R., & Pilote, C. (2012). Clients' preferred relationship approach with their financial statement auditor. Current Issues in Auditing, 6(1), 1e6. F€orordning om revisorer, SFS 1995:665 [Decree on accountants, 1995:665].

24. Glover, S. M., Prawitt, D. F., Wilks, T. J., & McDaniel, L. (2005). Why do auditors over-rely on weak analytical procedures? The role of outcome and precision. Auditing: A Journal of Practice & Theory, 24(Suppl.), 197e233

25. Haslam, S. A., Oakes, P. J., Reynolds, K. J., & Turner, J. C. (1999). Social identity salience and the emergence of stereotype consensus. Personality and Social Psychology Bulletin, 25, 809–818.

26. Haslam, S. A. (2001). Psychology in organisations: The social identity approach. London: Sage Publications.

27. Haynes, C. M., Jenkins, J. G., & Nutt, S. R. (1998). The relationship between client advocacy and audit experience: an exploratory analysis. Auditing: A Journal of Practice and Theory, 17(2), 88e104.

28. Herda, D., & Lavelle, J. (2013). Auditor commitment to privately held clients and its effect on value-added audit service. Auditing: A Journal of Practice & Theory, 32(1), 113e137.

29. Hogg, M. A., & Abrams, D. (1988). Social identifications: A social psychology of intergroup relations and group processes. London: Routledge.

30. Hogg, M. A. (2003). Social identity. In M. R. Leary, & J. P Tangney (Eds.), Handbook of self and identity (pp. 462e479). New York: The Guilford Press.

31. Hogg, M. A., van Knippenberg, D., & Rast, D. E., III (2012). Intergroup leadership in organizations: leading across group and organizational boundaries. Academy of Management Review, 37(2), 232e255.

32. Iyer, V. M. 1998. Characteristics of accounting firm alumni who benefit their former firm. Accounting Horizons 12(1): 18-30.

33. Iyer, V. M., E. M. Bamber, and R. Barefield. 1997. Identification of accounting firm alumni with their former firm: Antecedents and outcome. Accounting, Organizations and Society 22(3-4): 315-336.

34. Jenkins, J. G., & Lowe, D. J. (1999). Auditors as advocates for their clients: perceptions of the auditoreclient relationship. Journal of Applied Business Research, 15(2), 73e78.

35. Johnstone, K. M., M. H. Sutton, T. D. Warfield. 2001. Antecedents and consequences of independence risk: Framework for analysis. Accounting Horizons 15(1): 1-18.

36. Johnstone, K. M., Bedard, J. C., & Biggs, S. F. (2002). Aggressive client reporting: factors affecting auditors' generation of financial reporting alternatives. Auditing: A Journal of Practice & Theory, 21(1), 47e65

37. Kelley, T., & Margheim, L. (1987). The effect of audit billing arrangement on under-reporting of time and audit quality reduction acts. Advances in Accounting,5, 221e233.

38. King, R. R. 2002. An experimental investigation of self-serving biases in an auditing trust game: The effect of group affiliation. The Accounting Review 77(2): 265-284.

39. Kunda, Z. (1990). The case for motivated reasoning. Psychological Bulletin, 108(3), 480e498.

40. Kunda, Z. (1999). Social cognition: Making sense of people. Cambridge, MA: The MIT Press.

41. Lachman, R., and N. Aranya. 1986. Evaluation of alternative models of commitments and job attitudes of professionals. Journal of Occupational Behavior 7(3): 227-243.

42. Lambert, R., Leuz, C., & Verrecchia, R. E. (2007). Accounting information, disclosure, and the cost of capital. Journal of Accounting Research, 45(2), 385e420

43. Lembke, S., and M. G. Wilson. 1998. Putting the "team" into teamwork: Alternative theoretical contributions for contemporary management practice. Human Relations 51(7): 927-944.

44. Mael, F. A., and B. E. Ashforth. 1995. Loyal from day one: Biodata, organizational identification, and turnover among newcomers. Personnel Psychology 48(2): 309-333.

45. Mael, F. A., and B. E. Ashforth. 1992. Alumni and their alma mater: A partial test of the reformulated model of organizational identification. Journal of Organizational Behavior 13(2):103-123.

46. McNair, C. J. (1991). Proper compromises: the management control dilemma in public accounting and its impact on auditor behavior. Accounting, Organizations and Society, 16(7), 635e653

47. €Ohman, P., H€ackner, E., & S€orbom, D. (2012). Client satisfaction and usefulness to external stakeholders from an audit client perspective. Managerial Auditing Journal, 27(5), 477e499.

48. Otley, D. T., & Pierce, B. J. (1996a). Audit time budget pressure: consequences and antecedents. Accounting, Auditing & Accountability Journal, 9(1), 31e58.

49. Otley, D. T., & Pierce, B. J. (1996b). The operation of control systems in large audit firms. Auditing: A Journal of Practice & Theory, 15(2), 65e84

50. Perry, J. (Ed.). (2008). Personal identity, Vol. 2, Oakland, CA: University of California Press.

51. Pierce, B., & Sweeney, B. (2004). Cost-quality conflict in audit firms: an empirical investigation. European Accounting Review, 13(1), 415e441

52. Russo, T. C. 1998. Organizational and professional identification: A case of newspaper journalists. Management Communication Quarterly 12(1): 72-111.

53. Scott, S. G. 1997. Social identification effects in product and process development teams. Journal of Engineering Technology Management 14(2): 97-127.

54. Sedikides, C., & Brewer, M. B. (2001). Individual self, relational self, collective self. Philadelphia, PA: Psychology Press.

55. Shafer, W. E., Park, L. J., & Liao, W. M. (2002). Professionalism, organizational-professional conflict and work outcomes: a study of certified management accountants. Accounting, Auditing & Accountability Journal, 15(1), 46e68.

56. Shafer, W. E., & Wang, Z. (2010). Effects of ethical context on conflict and commitment among Chinese accountants. Managerial Auditing Journal, 25(4), 377e400.

57. Stefaniak, C. M., Houston, R. W., & Cornell, R. M. (2012). The effects of employer and client identification on internal and external auditors' evaluations of internal control deficiencies. Auditing: A Journal of Practice & Theory, 31(1), 39e56.

58. Towry, K. L. 2003. Control in a teamwork environment – The impact of social ties on the effectiveness of mutual monitoring contracts. The Accounting Review 78(4): 1069-1095.

59. Turner, J. C., Hogg, M. A., Oakes, P. J., Reicher, S. D., & Wetherell, M. S. (1987). Rediscovering the social group: A self categorization theory. Oxford: Blackwell

60. Valsiner, J. (2007). Culture in minds and societies: foundations of cultural psychology. New Delhi: Sage.

61. Wallace, J. E. 1995. Organizational and professional commitment in professional and nonprofessional organizations. Administrative Science Quarterly 40(2): 228-255.

62. Wan-Huggins, V. N., C. M. Riordan., and R. W. Griffeth. 1998. The development and longitudinal test of a model of organizational identification. Journal of Applied Social Psychology 28(8): 724-49.

63. Willett, C., & Page, M. (1996). A survey of time budget pressure and irregular auditing practices among newly qualified UK chartered accountants. British Accounting Review, 28(2), 101e120.

64. Wyatt, A. R. 2004. Accounting professionalism—They just don't get it! Accounting Horizons 18(1): 45-53.

Artificial intelligence and advanced technologies - some case studies from European research space

Daniela Gotseva,
Technicla University Sofia
Rumen Trifonov,
Technicla University Sofia
Pavel Stoynov,
Technicla University Sofia, Sofia University,
Perun R&D Academy, Sofia, Bulgaria

Abstract.
Being advanced technology under development itself, Artificial Intelligence may help for further development of many other advanced technologies in key areas of human activity. In the article, several case studies from European research space are considered illustrating the symbiosis between Artificial Intelligence and Advanced Technologies.

Key words: artificial intelligence, advanced technologies

Introduction

On 19 of April 2018, a conference named "HPC or Extreme Scale Scientific and Industrial Applications" took place at the National Palace of Culture in Sofia, Bulgaria. The event was a part of the Bulgarian Presidency of EU events and was organized in close cooperation with the European Commission. The event demonstrated the huge interest in the advanced technologies, including future and emerging technologies. Most often, these new technologies come together with Artificial Intelligence intended to monitor and manage the technological systems.

Artificial Intelligence (AI)

Artificial intelligence (AI) is a research field that studies how to realize the intelligent human behaviours on a computer. The ultimate goal of AI is to make a computer that can learn, plan, and solve problems autonomously. Although AI has been studied for more than half a century, we still cannot make a computer that is as intelligent as a human in all aspects. However, we do have many successful applications. In some cases, the computer equipped with AI technology can be even more intelligent than us. The Deep Blue system which defeated the world chess champion is a well-know example.

The main research topics in AI include: problem solving, reasoning, planning, natural language understanding, computer vision, automatic programming, machine learning, and so on. Of course, these topics are closely related with each other. For example, the knowledge acquired through learning can be used both for problem solving and for reasoning. In fact, the skill for problem solving itself should be acquired through learning. Also, methods for problem solving are useful both for reasoning and planning. Further, both natural language understanding and computer vision can be solved using methods developed in the field of pattern recognition.

Advanced Technologies (AT)

We may divide the advanced technologies into two groups – Advanced Information Technologies and Advanced Technologies of the Material World.

Among the Advanced Information Technologies today we may consider Supercomputing (High Performance Computing), Artificial Neural Networks and Quantum computing.

Among the Advanced Technologies of the Material World today we may consider Advanced Materials, New Energies, Automation and Robotics, Micro- and Nanotechnologies, Photonics, Industrial Biotechnologies, Military technologies, Aero and Space technologies, Cutting Edge Health Care technologies etc.

Below we will mention only some of the advanced technologies.

Supercomputing (High Performance Computing). Computing is increasingly integrated into all aspects of societies, economies and everyday life.

Nowdays, supercomputers are an essential tool for scientists and engineers. However, if the software used to solve complex scientific questions or engineering problems is not appropriately programmed or optimized, even powerful supercomputers can not generate quick results. To make efficient and sustainable use of High

Performance Computing (HPC), sound programming by well-trained and highly qualified staff is just as important as having state-of-the-art hardware.

Artificial Neural Networks. Neuromorphic computing platforms are brain-inspired computing devices which enable high-speed, low-energy simulations of spiking neural networks with synaptic plasticity.

Quantum computing. Quantum processors have a great potential for increasing the performance of computers. They are however still in its development phase. Quantum annealing should be mastered first, then quantum computing.

The first half of 2017 saw considerable news about the imminent impact of quantum computing (QC) on science, math and data analysis (Palmer, 2017). Diverse viewpoints predict from years to decades for initial impact, and debate what type of problems will first be affected.

The conceptual appeal of QC systems is the use of their computational power, which grows exponentially with the number of quantum bits (or qbits), to reduce the time spent executing combinatorial algorithms, which on classical computers grows exponentially with the number of variables.

The most advanced commercially available quantum computers are currently delivered by D-Wave Systems. The number of qbits in these systems has doubled every other year for the past six years, a trend that is expected to continue.

Contemporary quantum computers implement one of two computational models: the gate of circuit model (Feynman, 1986) or the quantum annealing model (Kadowaki and Nishimori, 1998).

A quantum annealing processor is a special-purpose device that natively solves the quadratic unconstrained binary oprtimization (QUBO) problem. The goal of QUBO is to find the $x_i \in \{0,1\}$ that minimize $x^T Qx$ for upper-triangular matrix Q with $Q_{ij} \in R$. The QUBO form is equivalent to the unconstrained binary quadratic programming (UBQP) problem and the Ising model.

One can think of a quantum annealer as a hardware implementation of simulated annealing that uses quantum effects – superpositioning, entanglement, and quantum tunneling – to reduce the likelihood of getting stuck in a local minimum. In both quantum and simulated annealers, finding the x_i that truly minimizes $x^T Qx$ is not guaranteed. Instead, the goal is normally to find good solution quickly. A D-Wave quantum computer can propose a solution of $x^T Qx$ in only 5μs even for $N > 2000$ (where the search space is greater than 2^{2000}) as supported by a D-Wave 2000Q system.

Mapping computational problems into QUBO form is a key challenge that all subsequently-described applications face.

Applications of quantum computing are as follows:

- Graph Partitioning. Computational optimizations as graph-theoretic problems is ubiquitous throughout mathematics, computer science, physics, chemistry, bioscience, machine learning, and complex systems. Many of these problems are NP-hard and therefore rely on heuristic solutions (Mniszewski, Ushijima-Mwesigwa, Negre, 2017).
- Traffic Flow Optimization. Certain parts of the real-world problem of optimizing traffic flow can be mapped into a form suitable for QA (Neukart et al., 2017).
- Finance. Here specific applications include multi-period mean-variance portfolio optimization, hierarchical risk parity feature selection for credit scoring, and tax loss harvesting.
- Programming. Only in the last year or two have interfaces emerged that are higher level than the system-dependent quantum machine intrusion exposed by D-Wave's low-level system interface

Automation and robotics. Robotic machines play an ever larger role in industrial production and elsewhere. Automation is the key for creating complex and precise machines, as well as machines capable to work autonomously at conditions where humans can't survive.

Artificial Intelligence and Advanced Technologies

Being advanced technology under development itself, Artificial Intelligence may help for further development of many other advanced technologies in key areas of human activity.

Artificial Intelligence can successfully be used in Information Technologies, for example for analysis of Big Data, for development of Cyber-security, for 3D visualization and Virtual Reality.

Virtual and Augmented Reality technologies are booming in Europe. They are expected to create 480 000 jobs, with the industry increasing in value to upwards of 34 billion euro by 2020. Virtual Reality can be applied across a wide array of sectors.

In manufacturing and mechanical engineering, an example of application of Virtual Reality is the product Oculus Medium – 3D modeling tool for creating virtual "sculptures" through Virtual Reality headsets and controllers.

In health care, examples of Virtual Reality are Fearless – application utilizing procedures based on exposure therapy for overcoming phobias – and Medical Realities – educational platform combining 360 degree videos with 3D animations.

In finance, Cisco Spark VR is an application for business collaboration and interaction.

In creative industries, Tilt brush is VR painting tool of Google, and Quill VR is VR painting tool of Facebook.

In construction, ScanARchitect is architectural and construction visualization tool.

Examples of using Virtual Reality technologies in tourism are Iceland and Hotel Abruzzi Rome.

In retail, example of Virtual reality application is Virtuosa VR – 360 degree video with VR catalogue of Virtuosa, a jewellery shop.

Artificial Intelligence also can successfully be used to develop Advanced Technologies of the Material World, for example Industrial Biotechnologies and Health Care technologies.

Below we will consider several case studies from European research space illustrating the symbiosis between Artificial Intelligence and Advanced Technologies.

Whatify. Whatify is awareness-raising campaign that stimulates the modernization of European industry. The campaign focuses on the technological transformation of traditional SMEs, promotion of regional digitization and uptake of advanced technologies – notably Key Enabling Technologies. Whatify seeks to boost the rate of European industrial modernization through increased productivity and efficiency, resulting in improved competitiveness and job creation. Watify promotes the technological transformation of a wide range of sectors, across a wide range of technologies.

The key enabling technologies are: advanced materials, micro and nanoelectronics, nanotechnology, industrial biotechnology, advanced manufacturing technologies and photonics.

The sectors transformed are: manufacturing, health care, finance, creative industries, mechanical engineering, construction, tourism, retail, agro-food.

Digitisation technologies are: automation and robotics, internet of things, big data, 3D visualization, artificial intelligence, high-performance computing – HPC, cybersecurity.

Watify presents 100 Success Stories identifying and documenting a pool of challenges and benefits that individual companies have faced in their technological transformation process. We consider here three of them: VR Express, Printivo and Redborder.

Printivo Ltd. Is a bioprinting company combining cutting-edge bioengineering technology and up-to-date scientific approach. Printivo presents Eve – desktop 3D bioprinter, which has been specialized for the creation of 3D human tissue grafts in any size and conformation.

VR Express is a virtual reality service provider, creating custom VR content ranging from 360 degree videos and VR animation to interactive experiences.

Redborder is an open source solution for traffic visibility, Big Data analytics, and dynamic Cybersecurity. It enables the creation of customizable dashboards and reports for corporate networks and Managed Service Providers. They implement Collect-Analyse-Act motto for cyberatacks, detection, the definition of KPIs to support Security Operational Centers.

PRACE. Partnership for Advanced Computing in Europe (PRACE) is an international non-profit organization with its seat in Brussels. The PRACE research Infrastructure provides a persistent world-class high performance computing service for scientists and researchers from academia and industry in Europe. The computer systems and their operations accessible through PRACE are provided and funded by five PRACE members (BSC representing Spain, CINECA representing Italy, CSCS representing Switzerland, GCS representing Germany and GENCI representing France).

Since 2008, PRACE has offered a diverse training program in HPC, including seasonal schools, workshops and industrial seminars. Additionally, the PRACE Training Portal (2018) provides access to video tutorial and an extensive range of training materials. Early in 2017, PRACE introduced two new educational opportunities: Massive Open Online Courses (MOOCs) and the CodeVault (2018), a core repository for training codes open to everyone worldwide.

Currently, two MOOCs are available: Supercomputing: Discover how supercomputers are powering scientific breakthroughs (2018) and Managing Big Data with R and Hadoop (2018)

The PRACE Advanced Training Centers (PATCs) have been in operation since 2012: The Barcelona Supercomputing Center (Spain), CINECA – Consorzio Interuniversitario (Italy), CSC – IT Center for Science Ltd. (Finland), EPCC – the Edinburgh Parallel Computing Center at the University of Edinburgh (UK), The Gaus Center for Supercomputing (Germany), La Maison de la Simulation (France).

Different research projects have been awarded research time on the supercomputers of PRACE: Modelling olfactory system (2018), Photo protection in plants (2018), Planetesimal formation in protoplanetary disks (2018), Modelling the epoch of reionization - Ross, Dixon, Iliev, Mellema (2017) and Sullivan, Iliev, Dixon (2018), Visualising tornadoes (2018), Investigating the optical properties of grapheme nanoribbons (2018) – Avisati, Lisi, Gargiani, Della Pia, De Luca, Pacile, Cardoso, Varsano, Prezzi, Ferretti and Beti (2017) and

Della Pia, Avvisati, Ourjini, Cardoso, Versano, Prezzi, Ferretti, Mariani, Betti (2016), Paradoxical activation in cancer treatment (2018) – Papaleo, Saladino, Lambrughi, Lindorff-Larsen, Gervasio, Nussinov (2016) and Kuzmanic, Sutto, Saladino, Gervasio, Nebreda, Orozco, (2017), Mixed convection in channel flows (2018) – Pirozzoli, Bernardini, Verzicco, Orlandi (2017) and Pirozzoli, Bernardini, Orlandi (2016), Corner vortex shedding in solid rocket motors (2018) – Lacassagne, Bridel-Bertolomeu, Riber, Cuenot, Casalis, Nicoud (2017) and Lacassagne, Riber, Cuenot, Nicoud (2017), Multiscale modeling of ionic conductors (2018) – Zguns, Ruban, Skorodumova (2017) and Nilsson, Vekilova, Hellman, Klarbring, Simak, Skorodumova (2016).

Human Brain Project (HBP). The Human Brain Project (HBP) is one of the Future and Emerging Technology Flagship initiatives funded by the European Commission. It is a ten-year initiative in medicine, neuroscience, and computing which brings together scientists and institutions from 20 nations across Europe, and has a strong element of international cooperation.

The HBP is following a unique, multi-disciplinary approach to accelerate brain research, brain medicine and brain-inspired technology. The infrastructure makes available a growing range of data, models, software tools and hardware capabilities to scientists and industry.

The HBP is creating an advanced ICT platform to support researchers studying the brain and its diseases, empower brain-inspired computing and drive technological development. It includes cloud-based collaborative virtual experiments, data analytics and compute services and databases that enable meta-data handling and provenance tracking. Leading-edge supercomputers, brain-inspired neuromorphic computers, and neurorobotic systems combining simulated brains with robotic bodies, are being provided to scientists. The HBP is developing advanced software for big data analytics, modeling and simulation at all levels of brain organization – from the level of single molecules to the whole brain – to understand how the different levels of brain organization interact and generate complex behavior. Researchers can access all these from their own laboratories and collaborate with other labs.

Whole brain is in cm scale, neuron is in μm to mm scale, synapse is in nm to μm scale. The biggest challenge in understanding the brain is the bridging of different scales in space and time. This means identifying how genetic, molecular and cellular organization is translated into brain functions such as cognition and intelligence. It also means deciphering how behavior, disease and environment may influence what happens at the molecular, genetic or cellular levels – and all of this over time, from millisecond responses to the whole lifespan. The databases generated at these levels of description are so large that neuroscience needs high-performance computing to connect the scales into a comprehensive picture. The ability to combine simulation and Big Data analytics with experiments is key to making brain complexity scientifically tractable.

The HBP's quest to understand the human brain is advancing well. Detailed online brain atlases and models are being created. New insights are emerging from a broad spectrum of experiments, supercomputing-based analysis of large, complex data sets and simulations of molecules, cells and networks of cells. Progress is being made in the area of neurorobotics, where brain models are tested by connecting them to physical or virtual robots.

The worldwide scientific community can now start exploring the initial versions of the HBP six ICT Platforms – HBP-ICT (2018).

The Platforms reflect the HBP's key objectives: to gather and disseminate data describing the brain, to simulate and build models of the brain, to develop brain-inspired computing and robotics, and to create a global scientific community around the developing research infrastructure.

The Platforms consist of prototype hardware, software tools, databases, programming interfaces, and initial datasets, which will be refined and expanded on an on-going basis in close collaboration with end users.

The Platforms are the result of an extensive multidisciplinary effort, involving more than 750 scientists and engineers from over 100 institutions.

The HBP Collaboratory collects tools from the HBP Platforms in one place and organizes them into collaborative workspaces. Users can create a workspace, build a team, describe a project, collect and organize tools, share data, use Jupyter notebooks to share ideas, code and workflows.

Similar projects are developed outside EU. Below is a short list of currently active similar projects. EU: The Human Brain Project, 2013; USA: The brain initiative, 2014; Australia: The Australian Brain Alliance, 2016; China: The China Brain Project, 2016; Japan: Brain/MINDS project, 2016; Canada: Government/Brain Canada joint funding, 2017.

The Neuroinformatics Platform – HBP-NIP (2018) serves as the Human Brain Project's search engine for distributed data, data repositories, brain atlases and knowledge about the brain. The Platform consists of APIs for querying and a web-based platform and application programming interface (APIs), i. e. a set of standards, protocols and tools for building software applications. Data can be examined by species, contributing laboratory, methodology, brain region and data type, thereby allowing functionality not currently available elsewhere. The data are predominantly organized into atlases (HBP Strategic Rodent Brain Atlases and HBP Hyman Brain Atlasses) and linked to the KnowledgeSpace – a collaborative community-based encyclopedia.

The Neurorobotics Platform – HBP-NRP (2018) – is an Internet-accessible simulation system that allows the simulation of robots and environments controlled by spiking neural network. The Platform enables simple virtual closed-up experiments in cognitive neuroscience to be performed using brain models developed within the HBP, with the capability to customize several variables, such as the environmental and physical parameters, using a Robot Designer, Environment Builder and a Closed Loop Engine.

Released in April 2016, the Human Brain Project's Brain Simulation Platform (BSP) – HBP-BSP (2018) - is one of the six internet-accessible ICT Platforms for collaborative brain research. The goal is to offer scientists user-friendly tools to reconstruct and simulate data-driven models of neurons and whole brain tissue. A key driver was the Blue Brain Project's work to reconstruct and simulate neocortical microcircuitry in somatosensory cortex, and the application of Blue Brain techniques and workflows to other brain regions, notably cerebellum, hippocampus and the basal ganglia.

The Platform provides reconstruction and simulation pipelines, packaged into web-accessible workflows or showcased as use cases.

Medical Informatics Platform of HBP – HBP-MIP (2018) is to provide researchers with the ability to access and analyze large amount of anonymised clinical data from hospital, research, and pharmaceutical clinical trial databases through an innovative data management system.

The system integrates heterogeneous data formats seamlessly and federates data sources into a harmonized virtual database with a customized interface for navigation and data mining. The patterns discovered in the data ("biological signatures" which uniquely identify diseases) generate new hypotheses about brain disease classifications based on biological, physiological and anatomical features, in addition to the classical patterns of phenomenology expressed in symptoms, signs and syndromes. In the long run, the MIP will unlock the wealth of information stored in medical and research databases will provide a credible and rapid path to precision (or personalized) medical care.

High Performance Analytics and Computing (HPAC) Platform of Human Brain Project – HBP-HPAC (2018) provides high performance computing, storage and data processing capabilities to run simulations of sophisticated, detailed brain models and to analyze large, complex datasets. The execution of some of the tools, for example for visualization and data management, does not necessarily require a supercomputer, but they can be used on standard computers and notebooks. Other software, like parallel programming frameworks, can be used on both types of architectures. Scientists who would like to use a supercomputer need experience in programming with languages like C, C++, Python or Fortran.

In the frames of Human Brain Project – HBP-NCP (2018), two complementary Large-scale Neuromorphic Computing Systems (NCS) are set-up in Heidelberg (the BrainScaleS system) and Manchester (the SpiNNaker system).

The BrainScaleS system is based on physical (analogue or mixed-signal) emulations of neurons, synapse and plasticity models with digital connectivity, running up to ten thousand times faster than real time.

The SpiNNaker system is based on numerical models running in real time on custom digital multicore chips using ARM architecture. Models and simulation experiments are described in a Pyton script using the PyNN API. Experiments can be submitted via browser through a web API (Python client available).

Red Border - Cybersecurity and Artificial Intelligence

Red Border offers the ultimate real-time Network Traffic Analysis (NTA) and Cybersecurity Platform based on Big Data and Open source.

Red Border characteristics include: Collecting Machine Data, Scaling Out Big Data, Active Cybersecurity, Analyses, Open Source, Wireless Extensions.

Red Border ecosystem includes: redborder Probes and redborderApps. The probes are related to SNORT Red Border Edition. Applications are related to Traffic, Mobility, Intrusion, Vault, Social.

Red Border Intrusion offers IDS/IPS functionalities, allowing to detect or prevent intrusions. To do this, redborder NG Firewall/IPS probes or probes from third parties compatible with Snort are deployed, which are responsible for collecting data. The probes also perform the traffic analysis and the application of the specific security policies of each implementation, in order to generate alarms (IDS) or block traffic (IPS) in an active Cybersecurity action. The platform and the redborder IPS probes holds several configuration modes: Red Border appliances, bare metal, virtual on-premise, virtual on-cloud.

Vault: SIEM (Security Information and Event Management). The redborder Vault probes can be any network equipment that provides a historic of its logs (Syslog) in order to monitor the behavior of the network and these devices. These logs are sent to a correlation engine, where they are analyzed (Deep Log Analysis) by machine learning algorithms and AI, being able to model the usual network traffic and generating alarms when possible deviations or outliers in the behavior are detected. It has the following capabilities:

1. Timestamping: all logs that arrive in the system are given a time mark which validates the exact moment in which the reception of the log occurred.

2. Hashing: In order to avoid the alteration and/or modification of the stored logs, all of the entries are signed with a light algorithm with the server's own digital certificate.

3. Complete data: logs are stored in two different ways – a reduced version in the OLAP database (for the later visualization of the data), and the original version in a distributed file system.

Conclusion

Artificial Intelligence will be integral part of the technological transformation of our society. In this sense, it will help to further development of society.

In European Union, some potential benefits of technological transformation are: 30% increase of productivity, growth 15% more if using advanced technologies, 22% higher revenues, 2 times more jobs, 1,5 million additional jobs for EU economy.

References

Avisati, G., S. Lisi, P. Gargiani, A. Della Pia, O. De Luca, D. Pacile, C. Cardoso, D. Varsano, D. Prezzi, A. Ferretti and M. Beti (2017) FePc adsorbtion on the moire' superstructure of grapheme intercalated with a Co layer, J. Phys.Chem. C in press, doi: 10.1021/acs.jpcc.6b09875

CodeVault (2018) gitlab.com/PRACE-4IP/CodeVault

Corner vortex shedding in solid rocket motors (2018) – www.cerfacts.fr

Della Pia, A., G. Avvisati, O. Ourjini, C. Cardoso, D. Varsano, D. Prezzi, A. Ferretti, C. Mariani, M. Betti (2016) Electronic structure Evolution during the Growthof Graphene Nanoribbons on Au(110), J. Chem. Phys. C 120, 7323 (2016) doi: 10.121/acs.jpcc.5b11884

Feynman, R. (1986) Quantum mechanical computers. Found. Phys., 16 (6), 507-531.

HBP-BSP (2018) https://www. humanbrainproject.eu/bsp

HBP-HPAC (2018) https://www. humanbrainproject.eu/hpac

HBP-ICT (2018) https://www.humanbrainproject.eu/platforms-overview

HBP-MIP (2018) https://www. humanbrainproject.eu/mip

HBP-NCP (2018) https://www.humanbrainproject.eu/ncp

HBP-NIP (2018) https://www. humanbrainproject.eu/nip

HBP-NRP (2018) https://www. humanbrainproject.eu/nrp

Investigating the optical properties of grapheme nanoribbons (2018) https://grphene-flagship.eu

Kadowaki, T. and H. Nishimori (1988) Quantum annealing in the transverse Ising model. Phys. Rev. E, 58, 5355-5363.

Kuzmanic, A., L. Sutto, G. Saladino, F. Gervasio, A. Nebreda, M. Orozco, (2017) Molecular mechanism of canonical activation of p38 MAP kinase e-Life 6:e22175.

Lacassagne, L., T. Bridel-Bertolomeu, E. Riber, B. Cuenot, G. Casalis, F. Nicoud (2017) Numerical study of lateral blowing impact on corner vortex shedding instability. Submitted to Physics of Fluids.

Lacassagne, L., E. Riber, B. Cuenot, F. Nicoud (2017) On the impact of aluminium particles on corner vortex shedding instabilities in solid rocket motors using Euler-Lagrange approach. Submitted to Flow, Turbulence and Combustion.

Managing Big Data with R and Hadoop (2018) www.futurelearn.com/courses/big-data-r-hadoop

Mixed convection in channel flows (2018) http://newton.dima.uniroma1.it/mixconv/

Mniszewski, S., H. Ushijima-Mwesigwa, C. Negre (2017). Graph partitioning using the D-Wave for electronicstructure problems. In Identifying Computational Methods for Early Benefit from Quantum Computing (minisymposium).SIAM Annual Meeting, Pitsburgh, PA, July 10-14. 2017.

Modelling olfactory system (2018) www.pa.ibf.cnr.it/personale/migliore/index.php

Neukart, F., D. Von Dollen, G. Compostella, C. Seidel, S. Yarkoni, B. Parney (2017) Traffic flowoptimization using a quantum annealer. Preprint, arXiv:1708.01625

Papaleo, E., G. Saladino, M. Lambrughi, K. Lindorff-Larsen, F. Gervasio, R. Nussinov (2016) The role of protein loops and linkers in conformational dynamics and Allostery Chemical Reviews, 116, 6391-6423.

Multiscale modeling of ionic conductors (2018) www.kth.se/profile/snv123.

Nilsson, J., O. Vekilova, O. Hellman, J. Klarbring, S. Simak, N. Skorodumova (2016) Ionic conductivity in Gd-doped CeO2: Ab initio color-diffusion nonequilibrium molecular dynamics study, Phys. Rev. B 93, 024102.

Palmer, J. (2017) here, There and Everywhere: Quantum technology is beginning to come into its own. TheEconomist, Technology quarterly http://www.economist.com/news/essays/21717782-quantum-technology-beginning-come-its-own.

Paradoxical activation in cancer treatment (2018)

http://www.ucl.ac.uk/chemistry/research/research-groups/group-folder/protein-dynamics-francesco-gervasio

Photo protection in plants (2018) http://orcid.org/0000-0001-8870-0850

Pirozzoli, S., M. Bernardini, R. Verzicco, P. Orlandi (2017) Mixed convection in turbulent channels with unstable stratification. J. Fluid Mech. 821, pp. 482-516.

Pirozzoli, S., M. Bernardini, P. Orlandi (2016) Passive scalars in turbulent channel flow at high Reynolds number, J. Fluid Mech.788, pp. 614-638.

PRACE Training Portal (2018) www.training.prace-ri.eu

Planetesimal formation in protoplanetary disks (2018) http://pencil-code.nordita.org/

Redborder.com

Ross, H. E., K. L. Dixon, I. T. Iliev, G. Mellema (2017) Simulating the impact of X-ray heating during the cosmic dawn. MNRAS, 468, 3785.

Sullivan, D., I. T. Iliev, K. L. Dixon (2018) Using artificial Neural Networks to Constarin the Halo Baryon Fraction during Reionization. MNRAS, 473, 38.

Supercomputing: Discover how supercomputers are powering scientific breakthroughs (2018)
www.futurelearn.com/courses/supercomputing

Visualising tornadoes (2018)
summerofhpc.prace-ri.eu/wp-content/uploads/2017/08/SoHPC2016_reports-1.pdf

Watify.en

www.price-ri.eu/SHAPE

Zguns, P., A. Ruban, N. Skorodumova (2017) Ordering and phase separation in Gd-doped ceria: a combined DFT, cluster expansion and Monte Carlo study, Phys. Chem, 19, 26606.

Additive processes with return to zero and Stoynov processes

Pavel Todorov Stoynov
Sofia University "St. Kilment Ohridski",
Institute of Mathematics and Informatics of Bulgarian Academy of Sciences,
Institute of Information Sciences of Perun R&D Academy, Sofia, Bulgaria

Abstract.

Two families of processes are considered in this article – additive processes with return to zero and Stoynov processes.

Additive processes with return to zero are a specifc kind of generalized additive processes.

Stoynov distributions and processes or also Switch Time distributions and processes (ST distributions and processes) can be considered as specific kind of generalized gamma distributions and their corresponding processes.

Here by switch time we denote the time of sharp change of value of a stochastic process (jump) or sharp change of some characteristics of a stochastic process (regime switch).

Key words: processes with return to zero, Switch-time family distributions and processes, Stoynov family distributions and processes

Introduction. Additive processes – characterization and possible generalisations

With a given filtered probability space (Ω, F, F_t, P) satisfying the usual conditions, an (one or d-dimensional) additive process is defined as a stochastic process $\{X(t), t \geq 0\}$ which is càdlàg and satisfies the following conditions:

1. $X(0) = 0$;

2. The process has independent increments, i. e. for every sequence of numbers

$$0 < t_1 < t_2 < \cdots < t_n$$

the random variables

$$X(t_1) - X(0), X(t_2) - X(t_1), \ldots, X(t_n) - X(t_{n-1})$$

are independent;

3. The process is stochastically continuous (continuous in probability), i. e.

$$\lim_{t \to s} X(t) = X(s),$$

where the limit is taken by probability.

The additive processes are introduced by Lévy (1937, 1992). Their properties are studied in Sato (1999). In Sato (1999), the following theorem is presented:

Theorem (Sato) Let $\{X(t), t \geq 0\}$ be an additive process with values in R^d. Then:

1. For every $t \geq 0$ the random variable $X(t)$ has infinitely divisible distribution.

2. The distribution of $\{X(t), t \geq 0\}$ is uniquely identified by its point characteristics $\{(A(t), \mu(t), \Gamma(t)), t \geq 0\}$, which are related to the characteristic function in the following way:

$$Ee^{iuX(t)} = e^{\psi(t,u)}$$

and

$$\psi(t,u) = -\frac{1}{2} uA(t)u + iu\Gamma(t) + \int_{R^d} \mu(t, dx)(e^{iux} - 1 - iux1_{\{|x| \leq 1\}}).$$

The point characteristics of the additive process satisfy the properties:

2.1. For every $t \geq 0$ $A(t)$ is positive definite matrix with dimension $d \times d$, $\Gamma(t)$ is a function with values which are vectors in R^d and $\mu(t, dx)$ is a positive measure in R^d, satisfying the conditions $\mu(t, \{0\}) = 0$ and $\int_{R^d} \min(|x|^2, 1)\mu(t, dx) < \infty$.

2.2. $A(0)=0, \mu(0,dx)=0, \Gamma(0)=0$ and for all $t \geq s \geq 0$ the matrix $A(t)-A(s)$ is a positive definite matrix with dimension $d \times d$ and $\mu(s,B) \leq \mu(t,B)$ for all measurable sets $B \in \mathrm{B}(R^d)$.

2.3. If $s \to t$, then $A(s) \to A(t), \Gamma(s) \to \Gamma(t), \mu(s,B) \to \mu(t,B)$ for all measurable sets $B \in \mathrm{B}(R^d)$, for which $B \subset \{x : |x| \geq \varepsilon\}$ for some $\varepsilon > 0$.

Conversely, for the family triples $\{(A(t), \mu(t), \Gamma(t)), t \geq 0\}$, which satisfies 2.1., 2.2. and 2.3. there is an additive process $\{X(t), t \geq 0\}$ with the corresponding point characteristics.

Important special cases of the additive processes are the processes for which there exists the following parametric representation of the point characteristics:

1. $A(t)=\int_0^t \sigma^2(s)ds$, where for every $t \geq 0$ the matrix $\sigma(t)$ is a real matrix with dimension $d \times n$ for which $\int_0^T \sigma^2(t)dt < \infty$ for fixed $T > 0$ such that $\sigma(t)$ is defined in the interval $[0,T]$.

2. $\mu(t,B)=\int_0^t \nu(s,B)ds$ for all measurable sets $B \in \mathrm{B}(R^d)$, where $\{\nu(t), t \in [0,T]\}$ is a family of Lévy measures satisfying the condition $\int_0^T \int_{R^d} \min(|x|^2, 1)\nu(t,dx)dt < \infty$.

3. $\Gamma(t)=\int_0^t \gamma(s)ds$, where $\gamma : [0,T] \to R$ is a deterministic function with finite variation.

For the triple $(\sigma^2(t), \nu(t), \gamma(t))$ we say that it defines the local characteristics of the additive process. The additive processes with local characteristics are semimartingales.

There are different possibilities for generalization of the additive processes.

One way of generalization is to introduce jumps in deterministic moments, as proposed by Kallsen (1998). In this way, the process is not already stochastically continuous and with stationary increments. To characterize such kind of processes, to the point and local characteristics there is a need to add new characteristics identifying the position and the size of these deterministic-time jumps.

The point characteristics become

$$\{(\Theta, K(t), A(t), \mu(t), \Gamma(t)), t \geq 0\},$$

where Θ is a discrete set giving the times of the jumps and $K(t)$ is conditional distribution of the jumps for which

$$K(t,G)=\begin{cases} \mu(t,G)+\varepsilon_0(G)(1-\mu(t,R)), t \in \Theta \\ 0, t \notin \Theta. \end{cases}$$

Here $\varepsilon_0(G)$ is the Dirac measure in the point 0.

In the case when local characteristics exist we have

$$\{(\Theta, K(t), A(t)=\int_0^t \sigma^2(s)ds,$$

$$\mu(t,G)=\int_0^t \nu(s,G)ds+\sum_{s \in \Theta \cap [0,t]} \mu(s,G)=\int_0^t \nu(s,G)ds+\sum_{s \in \Theta \cap [0,t]} K(s, G \setminus \{0\}),$$

$$\Gamma(t)=\int_0^t \gamma(s)ds+\sum_{s \in \Theta \cap [0,t]} \Delta\Gamma(s)=\int_0^t \gamma(s)ds+\sum_{s \in \Theta \cap [0,t]} \int x K(s,dx)), t \geq 0\},$$

and the local characteristics are:

$$\{(\Theta, K(t), \sigma^2(t), \nu(t), \gamma(t)), t \geq 0\}.$$

A next step of generalization is to allow stochastic characteristics- drift, volatility and jump intensity. The corresponding processes are considered for example in Grigelionis (1973).

In this case, the point characteristics of the process are

$$\{(\Theta, K(\omega,t), A(\omega,t), \mu(\omega,t), \Gamma(\omega,t)), t \geq 0\}$$

and the local characteristics (when they exist) are

$$\{(\Theta, K(\omega,t), \sigma^2(\omega,t), \nu(\omega,t), \gamma(\omega,t)), t \geq 0\}.$$

The obtained characterization allows to specify also some specific jumps in random times and to characterize them separately from the remaining jumps of the process. In this case, the point characteristics can be presented as:

$$\{(\Theta(\omega,t), K(\omega,t), A(\omega,t), \mu(\omega,t), \Gamma(\omega,t)), t \geq 0\}$$

where the random times of the specific jumps are represented by the point process $\{\Theta(\omega,t), t \geq 0\}.$

It is also possible to combine specific jumps in specific random times with jumps in deterministic moments. In this case, the parameters of the process (in the case when local characteristics exist) are:

$$\{(\Theta_1, \Theta_2(\omega,t), K_1(\omega,t), K_2(\omega,t), A(\omega,t) = \int_0^t \sigma^2(\omega,s)ds,$$

$$\mu(\omega,t,G) = \int_0^t \nu(\omega,s,G)ds + \sum_{s\in\Theta_1\cap[0,t]} K_1(\omega,s,G\setminus\{0\}) + \sum_{\theta_i<t} K_2(\omega,\theta_i,G\setminus\{0\}),$$

$$\Gamma(t) = \int_0^t \gamma(s)ds + \sum_{s\in\Theta_1\cap[0,t]} \int xK_1(\omega,s,dx) + \sum_{\theta_i<t} \int xK_2(\omega,\theta_i,dx)), t \geq 0\}.$$

Here $K_1(\omega,t)$ and $K_2(\omega,t)$ are the conditional distributions of the jumps in deterministic and specific random times correspondingly.

Additive processes with return to zero of type „jump" in deterministic moments

We consider processes $X(t) = G(t) + J(t),$ where $G(t)$ as an additive process, and $J(t)$ is a process of jumps which ensure the returns to zero.

Their point characteristics are given by

$$\{(\Theta, K(t,B), A(t), \mu(t,B) = \mu_A(t,B) + \sum_{s\in\Theta\cap[0,t]} K(s, B\setminus\{0\}),$$

$$\Gamma(t) = \Gamma_A(t) + \sum_{s\in\Theta\cap[0,t]} \int xK(s,dx)), t \geq 0\}.$$

The set Θ is a discrete set giving the moments of jumps. $K(t,B)$ is the conditional distribution of the jumps, for which we have

$$K(t,B) = \begin{cases} \mu(t,B) + \varepsilon_0(B)(1-\mu(t,R)), t \in \Theta \\ 0, t \notin \Theta. \end{cases}$$

Here $\varepsilon_0(B)$ is the Dirak measure in zero and $\mu_A(t,B)$ and $\Gamma_A(t)$ are the corresponding characteristics of the additive process $G(t).$

The return into zero is a constraint for the process $J(t)$ and leads to additional characterization of $K(t,B).$ So, we have the following lemma:

Lemma 1. For the additive processes with return to zero of type „jump" in deterministic moments, given by the set $\Theta,$ the conditional distributions of the jmps $K(t,B)$ is given by:

$$K(t,B) = \begin{cases} \varepsilon_{-X(t-)}(B), t \in \Theta \\ 0, t \notin \Theta. \end{cases}$$

Proof: We have $K(t,B) = P_{\Delta X(t)|F_{t-}}(B),$ i. e. $K(t)$ is the conditional distribution of the jumps. For the processes with returns to zero we have:

$$\Delta X = X(t) - X(t-) = \begin{cases} 0 - X(t-) = -X(t-) = J(t), t \in \Theta \\ G(t) - G(t-), t \notin \Theta. \end{cases}$$

□

Corrolary 1. At the points of Θ for processes with returns to zero we have

$$\sum_{s \in \Theta \cap [0,t]} K(s, B \setminus \{0\}) = \sum_{s \in \Theta \cap [0,t]} \varepsilon_{-X(s-)}(B \setminus \{0\}).$$

Proof: It foolows from Lemma 1 applied to the left part of the equation.. □

Corrolary 2. We have:

$$\mu(t, B) = \mu_A(t, B) + \sum_{s \in \Theta \cap [0,t]} \varepsilon_{-X(s-)}(s, B \setminus \{0\}).$$

Proof: It follows from

$$\mu(t, B) = \mu_A(t, B) + \sum_{s \in \Theta \cap [0,t]} K(s, B \setminus \{0\}) =$$

$$= \mu_A(t, B) + \sum_{s \in \Theta \cap [0,t]} \varepsilon_{-X(s-)}(s, B \setminus \{0\}).$$ □

Corrolary 3. We have the equation

$$\int xK(s, dx) = J(s).$$

Proof: We have

$$\int xK(s, dx) = \int x\varepsilon_{-X(s-)}(dx) = -X(s-) = J(s).$$ □

Corrolary 4. We have the equation:

$$\Gamma(t) = \Gamma_A(t) - \sum_{s \in \Theta \cap [0,t]} X(s-).$$

Proof: We have

$$\Gamma(t) = \Gamma_A(t) + \sum_{s \in \Theta \cap [0,t]} \Delta\Gamma(s) =$$

$$= \Gamma_A(t) + \sum_{s \in \Theta \cap [0,t]} \int xK(s, dx) = \square$$

$$= \Gamma_A(t) - \sum_{s \in \Theta \cap [0,t]} X(s-).$$

Corrolary 5. In the case where the additive processes with returns to zero of type "jump" are with local characteristics, the point characteristics are:

$$\{(\Theta, K(t, B), A(t) = \int_0^t \sigma^2(s)ds, \mu(t, B) =$$

$$= \int_0^t \nu(s, B)ds + \sum_{s \in \Theta \cap [0,t]} \varepsilon_{-X(s-)}(B \setminus \{0\}),$$

$$\Gamma(t) = \int_0^t \gamma(s)ds - \sum_{s \in \Theta \cap [0,t]} X(s-)), t \geq 0\}$$

and the local characteristics are

$$\{(\Theta, K(t, B), \sigma^2(t), \nu(t), \gamma(t)), t \geq 0\}.$$

Proof: We have

$$\{(\Theta, K(t,B), A(t) = \int_0^t \sigma^2(s)ds, \mu(t,B) =$$

$$= \int_0^t \nu(s,B)ds + \sum_{s\in\Theta\cap[0,t]} \mu(s,B) =$$

$$= \int_0^t \nu(s,B)ds + \sum_{s\in\Theta\cap[0,t]} K(s, B\setminus\{0\}) =$$

$$= \int_0^t \nu(s,B)ds + \sum_{s\in\Theta\cap[0,t]} \varepsilon_{-X(s-)}(B\setminus\{0\}),$$

$$\Gamma(t) = \int_0^t \gamma(s)ds + \sum_{s\in\Theta\cap[0,t]} \Delta\Gamma(s) = \int_0^t \gamma(s)ds - \sum_{s\in\Theta\cap[0,t]} X(s-)), t \geq 0\}.$$

The local characteristics are determined from here immediately. □

Corrolary 6. In the case of Levy process with returns to zero of type "jump" in deterministic moments the point characteristics are

$$\{(\Theta, K(t,B), A(t) = \sigma^2 t, \mu(t,B) = t\nu + \sum_{s\in\Theta\cap[0,t]} \mu(s,B) =$$

$$= t\nu + \sum_{s\in\Theta\cap[0,t]} K(s, B\setminus\{0\}) = t\nu + \sum_{s\in\Theta\cap[0,t]} \varepsilon_{-X(s-)}(B\setminus\{0\}),$$

$$\Gamma(t) = \gamma t + \sum_{s\in\Theta\cap[0,t]} \Delta\Gamma(s) = \gamma t - \sum_{s\in\Theta\cap[0,t]} X(s-)), t \geq 0\}$$

and the local characteristics are

$$\{(\Theta, K(t,B), \sigma^2, \nu, \gamma), t \geq 0\}.$$

Proof: It follows from Corrolary 5 and from the from the characteristics of the Levy processes. □

Corrolary 7. For Brownian motion without drift with returns to zero of type "jump" in deterministics moments the point characteristics are

$$\{(\Theta, K(t,B), A(t) = \sigma^2 t, \mu(t,B) = \sum_{s\in\Theta\cap[0,t]} \mu(s,B) =$$

$$= \sum_{s\in\Theta\cap[0,t]} K(s, B\setminus\{0\}) = \sum_{s\in\Theta\cap[0,t]} \varepsilon_{-X(s-)}(B\setminus\{0\}),$$

$$\Gamma(t) = \sum_{s\in\Theta\cap[0,t]} \Delta\Gamma(s) = \sum_{s\in\Theta\cap[0,t]} \int xK(s,dx) =$$

$$= -\sum_{s\in\Theta\cap[0,t]} X(s-)), t \geq 0\},$$

and the corresponding local characteristics are:

$$\{(\Theta, K(t,B), \sigma^2, 0, 0), t \geq 0\}.$$

Proof: It foolos from Corrolary 5 and from the characteristics of the Brownian motion. □

Corrolary 8. For Poisson process with returns to zero of type "jumps" in deterministic moments the point (integral) characteristics are

$$\{(\Theta, K(t,B), A(t) = 0,$$

$$\mu(t,B) = \lambda f t + \sum_{s\in\Theta\cap[0,t]} \mu(s,B) = \lambda f t + \sum_{s\in\Theta\cap[0,t]} \varepsilon_{-X(s-)}(B\setminus\{0\}),$$

$$\Gamma(t) = \sum_{s\in\Theta\cap[0,t]} \Delta\Gamma(s) = -\sum_{s\in\Theta\cap[0,t]} X(s-)), t \geq 0\},$$

and the corresponding local characteristics are

$$\{(\Theta, K(t,B), 0, f\lambda, 0), t \geq 0\}.$$

Proof: It follows from Corrolary 5 and from the characteristics of the Poisson process. □

Additive processes with random characteristics and returns to zero of type "jump" in deterministic moments

They are processes $X(t)=G(t)+J(t),$ where $G(t)$ is an additive process and $J(t)$ is the jump process ensuring the returns to zero having random (stochastic) characteristics for $G(t)$ and $J(t).$

The additive processes with stochastic characteristics and returns to zero of type "jump" in deterministic moments have the following integral characteristics:

$$\{(\Theta, K(\omega,t,B), A(\omega,t), \mu(\omega,t,B) =$$
$$= \mu_A(\omega,t,B) + \sum_{s\in\Theta\cap[0,t]} \mu(\omega,s,B),$$
$$\Gamma(\omega,t) = \Gamma_A(\omega,t) + \sum_{s\in\Theta\cap[0,t]} \int xK(\omega,s,dx)), t\geq 0\}.$$

The returns to zero are constraints for the process $J(t)$ and lead to additional charactiristization of $K(\omega,t,B),$ given by the following lemma:

Lemma 2. For the additive process with stochastic characteristics and returns to zero of type "jump" in deterministic moments given by the set $\Theta,$ the conditional distribution of the jumps $K(\omega,t,B)$ is:

$$K(\omega,t,B) = \begin{cases} \varepsilon_{-X(t-)}(\omega,B), t\in\Theta \\ 0, t\notin\Theta. \end{cases}$$

Proof: Similar to the proof of Lemma1. □

Corrolary 2.9. When for the additive processes with stochastic characteristics and retrns to zero of type "jump" in deterministic moments there exist also local characteristics, the integral characteristics are

$$\{(\Theta, K(\omega,t,B), A(\omega,t) = \int_0^t \sigma^2(\omega,s)ds, \mu(\omega,t,B) =$$
$$= \int_0^t \nu(\omega,s,B)ds + \sum_{s\in\Theta\cap[0,t]} \varepsilon_{-X(s-)}(B\setminus\{0\}),$$
$$\Gamma(\omega,t) = \int_0^t \gamma(\omega,s)ds - \sum_{s\in\Theta\cap[0,t]} X(s-)), t\geq 0\}.$$

and the local characteristics are:

$$\{(\Theta, K(\omega,t,B), \sigma^2(\omega,t), \nu(\omega,t,B), \gamma(\omega,t)), t\geq 0\}.$$

Proof: We have

$$\{(\Theta, K(\omega,t,B), A(\omega,t) = \int_0^t \sigma^2(\omega,s)ds, \mu(\omega,t,B) =$$
$$= \mu_A(\omega,t,B) + \sum_{s\in\Theta\cap[0,t]} \mu(\omega,s,B) =$$
$$= \int_0^t \nu(\omega,s,B)ds + \sum_{s\in\Theta\cap[0,t]} K(\omega,s,B\setminus\{0\}) =$$
$$= \int_0^t \nu(\omega,s,B)ds + \sum_{s\in\Theta\cap[0,t]} \varepsilon_{-X(s-)}(B\setminus\{0\}),$$
$$\Gamma(\omega,t) = \int_0^t \gamma(\omega,s)ds + \sum_{s\in\Theta\cap[0,t]} \int xK(\omega,s,dx) =$$
$$= \int_0^t \gamma(\omega,s)ds + \sum_{s\in\Theta\cap[0,t]} X(s-)), t\geq 0\}.$$

From here and from local characteristics we see the result immediately. □

Corrolary 10. In the case of Levy processes with stochastic characteristics and returns to zero of type "jump" in deterministic moments the integral characteristics are

$$\{(\Theta, K(\omega,t), A(\omega,t) = t\sigma^2(\omega), \mu(\omega,t,B) =$$
$$= t\nu(\omega,B) + \sum_{s\in\Theta\cap[0,t)} K(\omega,s,B\setminus\{0\}) =$$
$$= t\nu(\omega,B) + \sum_{s\in\Theta\cap[0,t)} \varepsilon_{-X(s-)}(B\setminus\{0\}),$$
$$\Gamma(\omega,t) = t\gamma(\omega) + \sum_{s\in\Theta\cap[0,t)} \int xK(\omega,s,dx) =$$
$$= t\gamma(\omega) - \sum_{s\in\Theta\cap[0,t)} X(s-)), t \geq 0\}$$

and the local characteristics are

$$\{(\Theta, K(\omega,t), \sigma^2(\omega,t) = \sigma^2(\omega), \nu(\omega,t,B)$$
$$= \nu(\omega,B), \gamma(\omega,t) = \gamma(\omega)), t \geq 0\}.$$

Proof: We use Corrolary 9 and the characteristics of the Levy processes. □

Corrolary 11. For the Brownian motion with stochastic characteristics nad returns to zero of type "jump" in deterministic moments the point characteristics are

$$\{(\Theta, K(\omega,t), A(\omega,t) = t\sigma^2(\omega) = t\sigma^2(\omega),$$
$$\mu(\omega,t,B) = \sum_{s\in\Theta\cap[0,t)} K(\omega,s,B\setminus\{0\}) = \sum_{s\in\Theta\cap[0,t)} \varepsilon_{-X(s-)}(B\setminus\{0\}),$$
$$\Gamma(\omega,t) = \sum_{s\in\Theta\cap[0,t)} \int xK(\omega,s,dx) = -\sum_{s\in\Theta\cap[0,t)} X(s-)), t \geq 0\}.$$

and the local characteristics are

$$\{(\Theta, K(\omega,t), A(\omega) = \sigma^2(\omega), \nu(\omega,B) = 0,$$
$$\gamma(\omega,t) = 0), t \geq 0\}.$$

Proof: It follows from Corrolary 9 and from the characteristics of the Brownian motion. □

Corrolary 12. For the Poisson process with returns to zero of type "jump" in deterministic moments and stochastic characteristics the point characteristics are

$$\{(\Theta, K(\omega,t), A(\omega,t) = 0,$$
$$\mu(\omega,t) = ft\lambda(\omega) + \sum_{s\in\Theta\cap[0,t)} K(\omega,s,B\setminus\{0\}) =$$
$$= ft\lambda(\omega) + \sum_{s\in\Theta\cap[0,t)} \varepsilon_{-X(s-)}(B\setminus\{0\}),$$
$$\Gamma(\omega,t) = \sum_{s\in\Theta\cap[0,t)} \int xK(\omega,t,dx) = -\sum_{s\in\Theta\cap[0,t)} X(s-)), t \geq 0\}$$

and the local characteristics are:

$$\{(\Theta, K(\omega,t), 0, \nu(\omega) = \lambda(\omega) f, 0), t \geq 0\}.$$

Proof: We use Corrolary 9 and the characteristics of the Poisson process. □

Additive processes with ctochastic characteristics and returns to zero of type "jump" in specific random times

At the additive processes with stochastic characteristics and returns to zero of type "jump" at specific random moments among the jumps of the processes we choose specific jumps which happenat moments defined by the point process $\{\Theta(\omega,t), t \geq 0\}$.

Here again the retirn to zero is a constraint for the process $J(t)$ and it gives additional characterization of $K(\omega,t)$. More precisely, we have the following lemma.

Lemma 3. For the additive processes with stochastic characteristics and returns to zero of type "jump" at specific random moments given by the point process $\{\Theta(\omega,t),t\geq 0\}$, the conditional distribution of the jumps is

$$K(\omega,t,B)=\begin{cases}\varepsilon_{-X(t-)}(\omega,B),t\in\Theta\\ 0,t\notin\Theta.\end{cases}$$

Proof: Similar to the proofs of Lemma 1 and Lemma 2. □

Corrolary 13. The additive processes with stochastic characteristics and returns to zero of type "jump" at specific random times have the following integral characteristics:

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t),\mu(\omega,t,B)=$$
$$=\mu_A(\omega,t,B)+\sum_{\theta_i<t}\varepsilon_{-X(\theta_i-)}(B\setminus\{0\}),\Gamma(\omega,t)=$$
$$=\Gamma_A(\omega,t)-\sum_{\theta_i<t}X(\theta_i-)),t\geq 0\},$$

where θ_i are the moments of jumps of the process $\{\Theta(\omega,t),t\geq 0\}$.

Proof: The result of Lemma 3 is directly replaced in the general characteristics of the process:

$$\{(\Theta(\omega,t),K(\omega,t,B),A(\omega,t),\mu(\omega,t,B)=$$
$$=\mu_A(\omega,t,B)+\sum_{\theta_i<t}K(\omega,\theta_i,B\setminus\{0\}),$$
$$\Gamma(\omega,t)=\Gamma_A(\omega,t)+\sum_{\theta_i<t}\int xK(\omega,s,dx).$$

We obtain

$$(2.48)\quad\{(\Theta(\omega,t),K(\omega,t),A(\omega,t),\mu(\omega,t,B)=$$
$$=\mu_A(\omega,t,B)+\sum_{\theta_i<t}K(\omega,\theta_i,B\setminus\{0\})=$$
$$=\mu_A(\omega,t,B)+\sum_{\theta_i<t}\varepsilon_{-X(\theta_i-)}(B\setminus\{0\}),\Gamma(\omega,t)=$$
$$=\Gamma_A(\omega,t)+\sum_{\theta_i<t}\int xK(\omega,s,dx)=$$
$$=\Gamma_A(\omega,t)-\sum_{\theta_i<t}X(\theta_i-)),t\geq 0\}.$$

□

Corrolary 14. In the case where for the additive processes with stochastic characteristics and returns to zero of type "jump" at specific random moments there exist also local characteristics, the point characteristics can b epresented as:

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=\int_0^t\sigma^2(\omega,s)ds,$$
$$\mu(\omega,t,B)=\int_0^t\nu(\omega,s,B)ds+\sum_{\theta_i<t}\varepsilon_{-X(\theta_i-)}(B\setminus\{0\}),$$
$$\Gamma(\omega,t)=\int_0^t\gamma(\omega,s)ds-\sum_{\theta_i<t}X(\theta_i-)),t\geq 0\}.$$

and the local characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),\sigma^2(\omega,t),\nu(\omega,t,B),\gamma(\omega,t)),t\geq 0\}.$$

Proof: We have

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=\int_0^t\sigma^2(\omega,s)ds,$$

$$\mu(\omega,t,B)=\int_0^t\nu(\omega,s,B)ds+\sum_{\theta_i<t}K(\omega,\theta_i,B\setminus\{0\})=$$

$$=\int_0^t\nu(\omega,s,B)ds+\sum_{\theta_i<t}\varepsilon_{-X(\theta_i-)}(B\setminus\{0\}),$$

$$\Gamma(\omega,t)=\int_0^t\gamma(\omega,s)ds+\sum_{\theta_i<t}\Delta\Gamma(\omega,\theta_i)=$$

$$=\int_0^t\gamma(\omega,s)ds-\sum_{\theta_i<t}X(\theta_i-)),t\geq 0\}.$$

From here the local characteristics are determined immediately. □

Corrolary 15. In the case of Levy processes with stochastic characteristics and returns to zero of type "jump" in random moments the point characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=t\sigma^2(\omega),$$

$$\mu(\omega,t,B)=t\nu(\omega,B)+\sum_{\theta_i<t}K(\omega,\theta_i,B)=$$

$$=t\nu(\omega,B)+\sum_{\theta_i<t}\varepsilon_{-X(\theta_i-)}(B\setminus\{0\}),$$

$$\Gamma(\omega,t)=t\gamma(\omega)+\sum_{\theta_i<t}\Delta\Gamma(\omega,\theta_i)=$$

$$=t\gamma(\omega,t)+\sum_{\theta_i<t}\int xK(\omega,\theta_i,dx)=$$

$$=t\gamma(\omega,t)-\sum_{\theta_i<t}X(\theta_i-)),t\geq 0\}$$

and the local characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),\sigma^2(\omega),\nu(\omega,B),\gamma(\omega)),t\geq 0\}.$$

Proof: We use Corrolary 14 and the characteristics of the Levy processes. □

Corrolary 16. In the case of Brownian motion with stochastic characteristics and returns to zero of type "jump" at random moments the point characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=t\sigma^2(\omega),$$

$$\mu(\omega,t,B)=\sum_{\theta_i<t}K(\omega,\theta_i,B\setminus\{0\})=$$

$$=\sum_{\theta_i<t}\varepsilon_{-X(\theta_i-)}(B\setminus\{0\}),$$

$$\Gamma(\omega,t)=\sum_{\theta_i<t}\Delta\Gamma(\omega,\theta_i)=\sum_{\theta_i<t}\int xK(\omega,\theta_i,dx)=$$

$$=-\sum_{\theta_i<t}\int X(\theta_i-)),t\geq 0\}$$

and the local characteristics are

$$\{(\Theta(\omega,t),K(\omega,t,B),\sigma^2(\omega),\nu(\omega,B)=0,\gamma(\omega)=0),t\geq 0\}.$$

Proof: We use 9 and the characteristics of the Brownian motion. □

Corrolary 17. In the case of Poisson process with stochastic characteristics and returns to zero of type "jump" at random moments the point characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=0,$$
$$\mu(\omega,t,B)=t\lambda(\omega)f(B)+\sum_{\theta_i<t}K(\omega,\theta_i,B\setminus\{0\})=$$
$$=t\lambda(\omega)f(B)+\sum_{\theta_i<t}\varepsilon_{-X(\theta_i)}(B\setminus\{0\}),$$
$$\Gamma(\omega,t)=\sum_{\theta_i<t}\int xK(\omega,\theta_i,dx)=-\sum_{\theta_i<t}X(\theta_i-)),t\geq 0\}$$

and the local characteristics are

$$\{(\Theta(\omega,t),K(\omega,t,B),A(\omega)=0,\nu(\omega,B)=$$
$$=\lambda(\omega)f(B),\gamma(\omega)=0),t\geq 0\}.$$

Proof: We use Corrolary 9 and the characteristics of the Poisson process.. □

Additive processes with returns to zero of type „termination" in deterministic moments

These are additive by parts processes $X(t)=\sum_i G_i(t-t_i)1_{[t_i,t_{i+1})}(t),$ where $G_i(t)$ are additive processes, t_i are moments of returns to zero, and $1_A(t)$ is the indicator of the event $A.$

The additive processes with return to zero of type „termination" at deterministic moments have the following characteristics:

$$\{(\Theta=\{t_i\},K(t),A(t)=\sum_i A_i(t-t_i)1_{[t_i,t_{i+1})}(t),$$
$$\mu(t,B)=\sum_i \mu_i(t-t_i,B)1_{[t_i,t_{i+1})}(t)+\sum_{s\in\Theta\cap[0,t]}K(s,B\setminus\{0\}),$$
$$\Gamma(t)=\sum_i \Gamma_i(t-t_i)1_{[t_i,t_{i+1})}(t)+\sum_{s\in\Theta\cap[0,t]}\int xK(s,dx)),t\geq 0\}.$$

Here $A_i(t),\ \mu_i(t,B)$ and $\Gamma_i(t)$ are the corresponding characteristics of the additive processes $G_i(t).$ The set Θ is a discrete set giving the times of jumps. $K(t)$ is the conditional distribution of the jumps.

The return to zero is related to additional characterization of $K(t).$ The characterization may be obtained similar to the previous cases.

When the additive processes with returns to zero of type „termination" are with local characteristics, the point characteristics are

$$\{(\Theta=\{t_i\},K(t,G),A(t)=\sum_i\int_0^t\sigma_i^2(s-t_i)1_{[t_i,t_{i+1})}(s)ds,$$
$$\mu(t,G)=\sum_i\int_0^t\nu_i(s-t_i)1_{[t_i,t_{i+1})}(s)ds+\sum_{s\in\Theta\cap[0,t]}\varepsilon_{-X(s-)}(G\setminus\{0\}),$$
$$\Gamma(t)=\sum_i\int_0^t\gamma_i(s-t_i)1_{[t_i,t_{i+1})}(s)ds-\sum_{s\in\Theta\cap[0,t]}X(s-)),t\geq 0\}$$

and the local characteristics are:

$$\{(\Theta,K(t,G),\sigma^2(t)=\sum_i\sigma_i^2(t-t_i)1_{[t_i,t_{i+1})}(t),$$
$$\nu(t)=\sum_i\nu_i(t-t_i)1_{[t_i,t_{i+1})}(t),$$
$$\gamma(t)=\sum_i\gamma_i(t-t_i)1_{[t_i,t_{i+1})}(t)),t\geq 0\}.$$

This is because we have

$$\{(\Theta=\{t_i\},K(t,G),A(t)=\int_0^t\sigma^2(s)ds=$$

$$=\sum_i\int_0^t\sigma_i^2(s-t_i)1_{[t_i,t_{i+1})}(s)ds,$$

$$\mu(t,G)=\int_0^t\nu(s,G)ds+\sum_{s\in\Theta\cap[0,t]}\mu(s,G)=$$

$$=\sum_i\int_0^t\nu_i(s-t_i)1_{[t_i,t_{i+1})}(s)ds+\sum_{s\in\Theta\cap[0,t]}K(s,G\setminus\{0\})=$$

$$=\sum_i\int_0^t\nu_i(s-t_i)1_{[t_i,t_{i+1})}(s)ds+\sum_{s\in\Theta\cap[0,t]}\varepsilon_{-X(s-)}(G\setminus\{0\}),$$

$$\Gamma(t)=\int_0^t\gamma(s)ds+\sum_{s\in\Theta\cap[0,t]}\Delta\Gamma(s)=$$

$$=\sum_i\int_0^t\gamma_i(s-t_i)1_{[t_i,t_{i+1})}(s)ds+\sum_{s\in\Theta\cap[0,t]}\int xK(s,dx)=$$

$$=\sum_i\int_0^t\gamma_i(s-t_i)1_{[t_i,t_{i+1})}(s)ds-\sum_{s\in\Theta\cap[0,t]}X(s-)),t\geq0\}$$

In the case of Levy process with return to zero of type “termination” the point characteristics are

$$\{(\Theta=\{t_i\},K(t),A(t)=\int_0^t\sigma^2(s)ds=\sum_i\int_0^t\sigma_i^2 1_{[t_i,t_{i+1})}(s)ds=$$

$$=\sum_{t_i<t}(\min(t,t_{i+1})-t_i)\sigma_i^2,$$

$$\mu(t,G)=\sum_i\int_0^t\nu_i 1_{[t_i,t_{i+1})}(s)ds+\sum_{s\in\Theta\cap[0,t]}K(s,G\setminus\{0\})=$$

$$=\sum_{t_i<t}\nu_i(\min(t,t_{i+1})-t_i)+\sum_{s\in\Theta\cap[0,t]}K(s,G\setminus\{0\}),$$

$$\Gamma(t)=\sum_i\int_0^t\gamma_i 1_{[t_i,t_{i+1})}(s)ds+\sum_{s\in\Theta\cap[0,t]}\int xK(s,dx)=$$

$$=\sum_{t_i<t}\gamma_i(\min(t,t_{i+1})-t_i)+\sum_{s\in\Theta\cap[0,t]}\int xK(s,dx)),t\geq0\}$$

and the local characteristics are

$$\{(\Theta=\{t_i\},K(t),\sigma^2(t)=\sum_i\sigma_i^2 1_{[t_i,t_{i+1})}(t),$$

$$\nu(t)=\sum_i\nu_i 1_{[t_i,t_{i+1})}(t),$$

$$\gamma(t)=\sum_i\gamma_i 1_{[t_i,t_{i+1})}(t),t\geq0\}.$$

For Brownian motion without drift with returns to zero of type “termination” we have the integral characteristics

$$\{(\Theta = \{t_i\}, K(t), A(t) = \sum_{t_i<t} (\min(t, t_{i+1}) - t_i)\sigma_i^2,$$

$$\mu(t, G) = \sum_{s\in\Theta\cap[0,t]} K(s, G \setminus \{0\}),$$

$$\Gamma(t) = \sum_{s\in\Theta\cap[0,t]} \int xK(s, dx)), t \geq 0\},$$

and the corresponding local characteristics are

$$\{(\Theta = \{t_i\}, K(t, G), \sigma^2(t) = \sum_i \sigma_i^2 1_{[t_i, t_{i+1})}(t),$$

$$\nu(t) = 0, \gamma(t) = 0), t \geq 0\}.$$

For Poisson process with return to zero of type "termination" in deterministic moments the integral characteristics are

$$\{(\Theta = \{t_i\}, K(t), A(t) = 0,$$

$$\mu(t, G) = \sum_i \lambda_i (\min(t, t_{i+1}) - t_i) f_i + \sum_{s\in\Theta\cap[0,t]} K(s, G \setminus \{0\}),$$

$$\Gamma(t) = \sum_{s\in\Theta\cap[0,t]} \int xK(s, dx)), t \geq 0\},$$

and the corresponding local characteristics are

$$\{(\Theta = \{t_i\}, K(t), \sigma^2(t) = 0, \nu(t) =$$

$$= \sum_i \lambda_i f_i 1_{[t_i, t_{i+1})}(t), \gamma(t) = 0), t \geq 0\}.$$

Additive processes with stochastic characteristics and returns to zero of type "termination" in deterministic moments

They are processes $X(t) = \sum_i G_i(t - t_i) 1_{[t_i, t_{i+1})}(t),$ where $G_i(t)$ are additive processes with stochastic characteristics $A_i(\omega, t), \mu_i(\omega, t, B)$ and $\Gamma_i(\omega, t)$ correspondingly, and t_i are the moments of returns to zero.

Additive processes with returns to zero of type "termination" in deterministic moments and stochastic characteristics have point characteristics:

$$\{(\Theta = \{t_i\}, K(\omega, t), A(\omega, t) = \sum_i A_i(\omega, t - t_i) 1_{[t_i, t_{i+1})}(t),$$

$$\mu(\omega, t, B) = \sum_i \mu_i(\omega, t - t_i, B) 1_{[t_i, t_{i+1})}(t) + \sum_{s\in\Theta\cap[0,t]} K(\omega, s, B \setminus \{0\}),$$

$$\Gamma(\omega, t) = \sum_i \Gamma_i(\omega, t - t_i) 1_{[t_i, t_{i+1})}(t) + \sum_{s\in\Theta\cap[0,t]} \int xK(\omega, s, dx)), t \geq 0\}.$$

The set Θ is a discrete set giving the moments of the jumps. $K(t)$ is the conditional distribution of the jumps.

Again the retrn to zero is related to additional characterization of $K(t).$

When the additive processes with retrns to zero of type "termination" are with local stochastic characteristics, the point characteristics are

$$\{(\Theta, K(\omega,t), A(\omega,t) = \sum_i \int_0^t \sigma_i^2(\omega, s-t_i) 1_{[t_i,t_{i+1})}(s) ds,$$

$$\mu(\omega,t,G) = \sum_i \int_0^t \nu_i(\omega, s-t_i) 1_{[t_i,t_{i+1})}(s) ds + \sum_{s\in\Theta\cap[0,t]} \varepsilon_{-X(s-)}(G\setminus\{0\}),$$

$$\Gamma(\omega,t) = \sum_i \int_0^t \gamma_i(\omega, s-t_i) 1_{[t_i,t_{i+1})}(s) ds - \sum_{s\in\Theta\cap[0,t]} \int X(s-)), t \ge 0\}$$

and the local characteristics are

$$\{(\Theta, K(\omega,t), \sigma^2(\omega,t) = \sum_i \sigma_i^2(\omega, t-t_i) 1_{[t_i,t_{i+1})}(t),$$

$$\nu(\omega,t) = \sum_i \nu_i(\omega, t-t_i) 1_{[t_i,t_{i+1})}(t),$$

$$\gamma(\omega,t) = \sum_i \gamma_i(\omega, t-t_i) 1_{[t_i,t_{i+1})}(t)), t \ge 0\}.$$

Proof: We have

$$\{(\Theta, K(\omega,t), A(\omega,t) = \int_0^t \sigma^2(\omega,s) ds = \sum_i \int_0^t \sigma_i^2(\omega, s-t_i) 1_{[t_i,t_{i+1})}(s) ds,$$

$$\mu(\omega,t,G) = \int_0^t \nu(\omega,s,G) ds + \sum_{s\in\Theta\cap[0,t]} \mu(\omega,s,G) =$$

$$= \sum_i \int_0^t \nu_i(\omega, s-t_i) 1_{[t_i,t_{i+1})}(s) ds + \sum_{s\in\Theta\cap[0,t]} K(\omega,s,G\setminus\{0\}) =$$

$$= \sum_i \int_0^t \nu_i(\omega, s-t_i) 1_{[t_i,t_{i+1})}(s) ds + \sum_{s\in\Theta\cap[0,t]} \varepsilon_{-X(s-)}(G\setminus\{0\}),$$

$$\Gamma(\omega,t) = \int_0^t \gamma(\omega,s) ds + \sum_{s\in\Theta\cap[0,t]} \Delta\Gamma(\omega,s) =$$

$$= \sum_i \int_0^t \gamma_i(\omega, s-t_i) 1_{[t_i,t_{i+1})}(s) ds + \sum_{s\in\Theta\cap[0,t]} \int xK(\omega,s,dx) =$$

$$= \sum_i \int_0^t \gamma_i(\omega, s-t_i) 1_{[t_i,t_{i+1})}(s) ds - \sum_{s\in\Theta\cap[0,t]} \int X(s-)), t \ge 0\}$$

In the case of Levy processes with returns to zero of type “termination” and stochastic characteristics the point characteristics are

$$\{(\Theta, K(\omega,t), A(\omega,t) = \int_0^t \sigma^2(\omega,s)ds =$$

$$= \sum_i \int_0^t \sigma_i^2(\omega) 1_{[t_i,t_{i+1})}(s)ds =$$

$$= \sum_{t_i<t} (\min(t,t_{i+1}) - t_i)\sigma_i^2(\omega),$$

$$\mu(\omega,t,G) = \sum_i \int_0^t \nu_i(\omega) 1_{[t_i,t_{i+1})}(s)ds + \sum_{s\in\Theta\cap[0,t]} K(\omega,s,G\setminus\{0\}) =$$

$$= \sum_{t_i<t} \nu_i(\omega)(\min(t,t_{i+1}) - t_i) + \sum_{s\in\Theta\cap[0,t]} K(\omega,s,G\setminus\{0\}),$$

$$\Gamma(\omega,t) = \sum_i \int_0^t \gamma_i(\omega) 1_{[t_i,t_{i+1})}(s)ds + \sum_{s\in\Theta\cap[0,t]} \int xK(\omega,s,dx) =$$

$$= \sum_i \gamma_i(\omega)(\min(t,t_{i+1}) - t_i) + \sum_{s\in\Theta\cap[0,t]} \int xK(\omega,s,dx)), t \ge 0\},$$

and the local characteristics are

$$\{(\Theta, K(\omega,t), \sigma^2(\omega,t) = \sum_i \sigma_i^2(\omega) 1_{[t_i,t_{i+1})}(t),$$

$$\nu(\omega,t) = \sum_i \nu_i(\omega) 1_{[t_i,t_{i+1})}(t),$$

$$\gamma(\omega,t) = \sum_i \gamma_i(\omega) 1_{[t_i,t_{i+1})}(t)), t \ge 0\}.$$

For Brownian motion without drift and returns to zero of type “termination” and stochastic characteristics the point characteristics are

$$\{(\Theta, K(\omega,t), A(\omega,t) = \sum_{t_i<t} (\min(t,t_{i+1}) - t_i)\sigma_i^2(\omega),$$

$$\mu(\omega,t,G) = \sum_{s\in\Theta\cap[0,t]} K(\omega,s,G\setminus\{0\}),$$

$$\Gamma(\omega,t) = \sum_{s\in\Theta\cap[0,t]} \int xK(\omega,s,dx)), t \ge 0\},$$

and the corresponding local characteristics are

$$\{(\Theta, K(\omega,t), \sigma^2(\omega,t) = \sum_i \sigma_i^2(\omega) 1_{[t_i,t_{i+1})}(t),$$

$$\nu(\omega,t) = 0, \gamma(\omega,t) = 0), t \ge 0\}.$$

For the Poisson process with returns to zero of type “termination” in deterministic moments and stochastic characteristics the integral characteristics are

$$\{(\Theta, K(\omega,t), A(\omega,t) = 0,$$

$$\mu(\omega,t,G) = \sum_i \lambda_i(\omega)(\min(t,t_{i+1}) - t_i) f_i + \sum_{s\in\Theta\cap[0,t]} K(\omega,s,G\setminus\{0\}),$$

$$\Gamma(\omega,t) = \sum_{s\in\Theta\cap[0,t]} \int xK(\omega,s,dx)), t \ge 0\},$$

an d the corresponding local characteristics are

$$\{(\Theta, K(\omega,t), \sigma^2(\omega,t) = 0, \nu(\omega,t) =$$

$$= \sum_i \lambda_i(\omega) f_i 1_{[t_i,t_{i+1})}(t), \gamma(\omega,t) = 0), t \ge 0\}.$$

Additive processes with stochastic characteristics and returns to zero of type "termination" at specific random times

The additive processes with stochastic characteristics and returns to zero of type "termination" at specific random times T_i we have the following point characteristics:

$$\{(\Theta(\omega,t)=\{T_i\},K(\omega,t),A(\omega,t)=$$
$$=\sum_i A_i(\omega,t-T_i)1_{[T_i,T_{i+1})}(t),$$
$$\mu(\omega,t,B)=\sum_i \mu_i(\omega,t-T_i,B)1_{[T_i,T_{i+1})}(t)+\sum_{T_i<t} K(\omega,T_i,B\setminus\{0\}),$$
$$\Gamma(\omega,t)=\sum_i \Gamma_i(\omega,t-T_i)1_{[T_i,T_{i+1})}(t)+\sum_{T_i<t_i}\int xK(\omega,T_i,dx)),t\geq 0\}.$$

Here $\{\Theta(\omega,t),t\geq 0\}$ is a point random process, giving the moments of jumps T_i, and $K(\omega,t)$ is the conditional distribution of the jumps for which we have

$$K(\omega,t,G)=\begin{cases}\mu(\omega,t,G)+\varepsilon_0(G)(1-\mu(\omega,t,R)),t\in\Theta\\ 0,t\notin\Theta.\end{cases}$$

In this case, some specifc jumps in times T_i are selected among the jumps and defined by the process $\{\Theta(\omega,t),t\geq 0\}$.

The returns to zero is related to additional characterization of $K(\omega,t)$.

When the additive processes with returns to zero of type "termination" and specific jumps at random times are with local stochastic characteristics, the integral characteristics

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=$$
$$=\sum_{T_i<t}\int_{0_i}^{t}\sigma_i^2(\omega,s-T_i)1_{[T_i,T_{i+1})}(s)ds,$$
$$\mu(\omega,t,G)=\sum_{T_i<t}\int_{0_i}^{t}\nu_i(\omega,s-T_i)1_{[T_i,T_{i+1})}(s)ds+$$
$$+\sum_{T_i<t}K(\omega,T_i,G\setminus\{0\}),$$
$$\Gamma(\omega,t)=\sum_{T_i<t}\int_{0}^{t}\gamma_i(\omega,s-T_i)1_{[T_i,T_{i+1})}(s)ds+$$
$$+\sum_{T_i<t}\int xK(\omega,T_i,dx)),t\geq 0\}$$

and the local characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),\sigma^2(\omega,t)=\sum_{T_i<t}\sigma_i^2(\omega,t-T_i)1_{[T_i,T_{i+1})}(t),$$
$$\nu(\omega,t)=\sum_{T_i<t}\nu_i(\omega,t-T_i)1_{[T_i,T_{i+1})}(t),$$
$$\gamma(\omega,t)=\sum_{T_i<t}\gamma_i(\omega,t-T_i)1_{[T_i,T_{i+1})}(t)),t\geq 0\}.$$

Indeed, we have

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=\int_0^t\sigma^2(\omega,s)ds=$$

$$=\sum_{T_i<t}\int_{0_i}^t\sigma_i^2(\omega,s-T_i)1_{[T_i,T_{i+1})}(s)ds,$$

$$\mu(\omega,t,G)=\int_0^t\nu(\omega,s,G)ds+\sum_{T_i<t}K(\omega,T_i,G\setminus\{0\})=$$

$$=\sum_{T_i<t}\int_{0_i}^t\nu_i(\omega,s-T_i)1_{[T_i,T_{i+1})}(s)ds+\sum_{T_i<t}K(\omega,T_i,G\setminus\{0\}),$$

$$\Gamma(\omega,t)=\int_0^t\gamma(\omega,s)ds+\sum_{T_i<t}\Delta\Gamma(\omega,T_i)=$$

$$=\sum_{T_i<t}\int_0^t\gamma_i(\omega,s-T_i)1_{[T_i,T_{i+1})}(s)ds+$$

$$+\sum_{T_i<t}\int xK(\omega,T_i,dx)),t\geq 0\}$$

In the case of Levy processes with stochastic characteristics and returns to zero of type “termination” at specific random moments the point characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=\sum_{T_i<t}(\min(t,T_{i+1})-T_i)\sigma_i^2(\omega),$$

$$\mu(\omega,t,G)=\sum_{T_i<t}\nu_i(\omega)(\min(t,T_{i+1})-T_i)+\sum_{T_i<t}K(\omega,T_i,G\setminus\{0\}),$$

$$\Gamma(\omega,t)=\sum_{T_i<t}\gamma_i(\omega)(\min(t,T_{i+1})-T_i)+\sum_{T_i<t}\int xK(\omega,T_i,dx)),t\geq 0\}$$

and the local characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),\sigma^2(\omega,t)=\sum_{T_i<t}\sigma_i^2(\omega)1_{[T_i,T_{i+1})}(t),$$

$$\nu(\omega,t)=\sum_{T_i<t}\nu_i(\omega)1_{[T_i,T_{i+1})}(t),$$

$$\gamma(\omega,t)=\sum_{T_i<t}\gamma_i(\omega)1_{[T_i,T_{i+1})}(t)),t\geq 0\}.$$

For Brownian motion without drift and returns to zero of type “termination” at specific random moments and with stochastic characteristics the point characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=$$

$$=\sum_{T_i<t}(\min(t,T_{i+1})-T_i)\sigma_i^2(\omega),$$

$$\mu(\omega,t,G)=\sum_{T_i<t}K(\omega,T_i,G\setminus\{0\}),$$

$$\Gamma(\omega,t)=\sum_{T_i<t}\int xK(\omega,T_i,dx)),t\geq 0\},$$

and the local characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),\sigma^2(\omega,t)=\sum_{T_i<t}\sigma_i^2(\omega)1_{[T_i,T_{i+1})}(t),$$

$$\nu(\omega,t)=0,\gamma(\omega,t)=0),t\geq 0\}.$$

For the Poisson process with retrns to zero of type “termination” in specific random moments and stochastic characteristics the point characteristics are

$$\{(\Theta(\omega,t),K(\omega,t),A(\omega,t)=0,$$
$$\mu(\omega,t,G)=\sum_{T_i<t}\lambda_i(\omega)(\min(t,T_{i+1})-T_i)f_i+$$
$$+\sum_{T_i<t}K(\omega,T_i,G\setminus\{0\}),$$
$$\Gamma(\omega,t)=\sum_{T_i<t}\int xK(\omega,T_i,dx)),t\geq 0\},$$

and the corresponding local characteristics are:

$$\{(\Theta(\omega,t),K(\omega,t),\sigma^2(\omega,t)=0,\nu(\omega,t)=$$
$$=\sum_{T_i<t}\lambda_i(\omega)f_i 1_{[T_i,T_{i+1})}(t),\gamma(\omega,t)=0),t\geq 0\}.$$

Switch Time distributions (Stoynov distributions) - STF(n,β)

We say that a random variable ξ with probability mass function $f_\xi(x)$ has distribution of $ST(n,\beta)$ family and denote this fact $\xi\in STF(n,\beta),$ if the probability mass function of ξ is given by the formula

$$f_\xi(x)=\begin{cases}\sum_{k=1}^{n+1}P(D^n=k)f_\xi(x\mid D^n=k)=\sum_{k=1}^{n+1}P(D^n=k)f_{G^k}(x),x\geq 0\\ 0,x<0.\end{cases}$$

where G^k are random variables with probability mass function $f_{G^k}(x)=f(k,\beta)$ and D^n are random variables taking values $k=1,\ldots,(n+1)$ with probabilities $P(D^n=k)=\frac{C(n,\beta)n!}{\beta^k(n-k+1)!},k=1,\ldots,(n+1)$ where the coefficients $C(n,\beta)$ are given by the formulas:

$$C(n,\beta)=\frac{1}{I(n,\beta)},$$
$$I(0,\beta)=\frac{1}{\beta},$$
$$I(n,\beta)=\frac{1}{\beta}+\frac{n}{\beta}I(n-1,\beta),n=1,2,\ldots$$

Also, variables $\tilde{D}^n=D^n-1$ can be introduced taking values $k=0,\ldots,n$ with probabilities

$$P(\tilde{D}^n=k)=\frac{C(n,\beta)n!}{\beta^{k+1}(n-k)!},k=0,\ldots,n.$$

Then the probability mass function $f_\xi(x)$ of ξ may be presented also by the formula

$$f_\xi(x)=\begin{cases}\sum_{k=0}^{n}P(\tilde{D}^n=k)f_\xi(x\mid\tilde{D}^n=k)=\sum_{k=0}^{n}P(\tilde{D}^n=k)f_{G^{k+1}}(x),x\geq 0\\ 0,x<0.\end{cases}$$

Here for G^k we may adopt different families of distribution. For example, if we take $G^k\in\Gamma(k,\frac{1}{\beta})\equiv Erlang(k,\frac{1}{\beta}),$ i. e. $\xi\mid D^n\equiv Erlang(D^n,\frac{1}{\beta}),$ we obtain a distribution which we will call $ST1(n,\beta)$ distribution, having probability density function

$$f_{\xi}(x)=\begin{cases}C(n,\beta)e^{-\beta x}(1+x)^n,\ x\geq 0\\ 0, x<0\end{cases}.$$

Indeed, we have:

$$C(n,\beta)e^{-\beta x}(1+x)^n = C(n,\beta)e^{-\beta x}\sum_{k=0}^{n}\binom{n}{k}x^k = \sum_{k=0}^{n}C(n,\beta)\binom{n}{k}x^k e^{-\beta x} =$$

$$=\sum_{k=0}^{n}\frac{C(n,\beta)n!x^k e^{-\beta x}}{k!(n-k)!}=\sum_{k=0}^{n}\frac{C(n,\beta)n!\beta^{k+1}x^k e^{-\beta x}}{\beta^{k+1}(n-k)!k!}=$$

$$=\sum_{k=0}^{n}\frac{C(n,\beta)n!}{\beta^{k+1}(n-k)!}\frac{\beta^{k+1}x^k e^{-\beta x}}{k!}=$$

$$=\sum_{k=0}^{n}P(D^n=k+1)f_{G^{k+1}}(x)=$$

$$=\sum_{k=1}^{n+1}\frac{C(n,\beta)n!\beta^k x^{k-1}e^{-\beta x}}{\beta^k(n-k+1)!(k-1)!}=$$

$$=\sum_{k=1}^{n+1}\frac{C(n,\beta)n!}{\beta^k(n-k+1)!}\frac{\beta^k x^{k-1}e^{-\beta x}}{\Gamma(k)}=\sum_{k=1}^{n+1}\frac{C(n,\beta)n!}{\beta^k(n-k+1)!}\frac{\beta^k x^{k-1}e^{-\beta x}}{(k-1)!}=$$

$$\sum_{k=1}^{n+1}P(D^n=k)f_{G^k}(x).$$

Special cases of $ST1(n,\beta)$ distribution are $ST1(0,\beta)\equiv Exp(\beta)$ and $ST1(1,\beta)\equiv Lindley(\beta).$ The case $n=2$ is introduced in Stoynov (2010) and further generalized in some other publications of the author, for example Stoynov (2011) where Moment Generating Function of $ST1(2,\beta)$ is derived.

As another example we may consider negative binomial distribution for G^k, i. e. $G^k\in NB(k,e^{-\beta}).$ In this case, $\xi\mid D^n\equiv NB(D^n,e^{-\beta})$ and we obtain a distribution which we will call $ST2(n,\beta)$ distribution.

Special cases of $ST2(n,\beta)$ distribution are $ST2(0,\beta)\equiv NB(1,e^{-\beta})$ and $ST2(1,\beta)\equiv\frac{\beta}{\beta+1}NB(1,e^{-\beta})+\frac{1}{\beta+1}NB(2,e^{-\beta}).$

We say that a random variable ξ has a negative binomial distribution (Pólya distribution) and write this $\xi\in NB(r,p)$, if its probability satisfies

$$P(\xi=k)=\binom{r+k-1}{k}p^r(1-p)^k,\ k=0,1,2,\ldots$$

We may interpret this random variable as a counter of the failures till the $r-th$ success in a sequence of Bernoulli trials.

As a third example we may consider $G^k\equiv\delta_k(x)$ - random variable which takes value k with probability one. Then $\xi\equiv D^n\in D^n(\beta).$ In this case, we say that random variable ξ has $ST3(n,\beta)$ distribution and denote $\xi\in ST3(n,\beta).$

Special cases of $ST3(n,\beta)$ distribution are $ST3(0,\beta)\equiv D^0\equiv 1$ and $ST3(1,\beta)\equiv D^1.$

ST(n,β) processes

We say that a process $X(t)$ is $ST1(n,\beta)$ ($ST2(n,\beta)$) and denote this fact $X(t)\in ST1(t;n,\beta)$ ($X(t)\in ST1(t;n,\beta)$), if it has the following properties:

1) $X(0)=0.$

2) The process has independent increments, i. e. for

$$0 < t_1 < t_2 < \cdots < t_n$$

the random variables

$$X(t_1) - X(0), X(t_2) - X(t_1), \ldots, X(t_n) - X(t_{n-1})$$

are independent.

3) The process is continuous stochastically (continuous in probability), i. e.

$$\lim_{t \to s} X(t) = X(s),$$

where the limit is taken in probability.

4) $X(t)$ is pure jump process with jumps at times $T_{i'}, i = 1,2,\ldots$ and jump sizes $\Delta X(T_i) = 1.$

5) The intervals between two jumps are

$$\tau = T_i - T_{i-1} \in ST1(n-1, \beta)\,(ST2(n-1, \beta)), i = 0,1,\ldots, T_0 = 0$$

The first three conditions mean that $ST1(n, \beta)$ ($ST2(n-1, \beta)$) process is an additive process. If we replace condition 4) with condition

4') $X(t)$ is pure jump process with jumps at times $T_{i'}, i = 1,2,\ldots$ and jump sizes $\Delta X(T_i) = Y_i$, where Y_i are independent and identically distributed random variables,

we say that $X(t)$ is compound $ST1(n, \beta)$ ($ST2(n, \beta)$) process.

It is interesting to see for $ST(n, \beta)$ processes the distribution of the number of jumps till time t which we denote $N^n(t).$

We start with $ST1(n, \beta)$ process. When $X(t) \in ST1(t; n, \beta)$, we consider first the case $X(t) \in ST1(t;1, \beta)$. In this case, $\tau = T_i - T_{i-1} \in ST1(0, \beta) \equiv Exp(\beta), i = 0,1,\ldots, T_0 = 0$. In this case the process $N^1(t)$ is a Poisson process $N(t)$ and for it we have

$$P(N^1(t) = k) = \frac{(\beta t)^k}{k!} e^{-\beta t}.$$

In the case when $X(t) \in ST1(t; n, \beta)$ the intervals between two jumps are

$$\tau = T_i - T_{i-1} \in ST1(n-1, \beta) \equiv Erlang(D^n, \beta), i = 0,1,\ldots, T_0 = 0.$$

First we will consider the case where

$$\tau = T_i - T_{i-1} \in Erlang(n, \beta), i = 0,1,\ldots, T_0 = 0.$$

In this case, we have that the process which counts the jumps of an Erlang process till time t which we denote $N^E(t)$ is a Poisson process $N(t)$ with omitted $n-1$ jumps and actual jumps at every $n-$th jump of the Poison process, so

$$P(N^E(t) = k) = P(N(t) = kn) = \frac{(\beta t)^{kn}}{(kn)!} e^{-\beta t}.$$

So, in the case $X(t) \in ST1(t; n, \beta)$ we have

$$P(N^n(t) = k) = P(N(t) = kn) = \frac{(\beta t)^{kn}}{(kn)!} e^{-\beta t}.$$

Simulation of ST(n,β) distribution

The simulation of $ST(n, \beta)$ distribution and corresponding PMF graphics for different values of the parameters n and β can be done by using R language. Here is the code for creating graphic representation of the PMF (for $n = 100$ and $\beta = 2$):

```
#ST(n,beta) graphics
beta<-2
n<-100
```

```
In_beta<-function(n,beta, recursive=TRUE)
{if (n==0) return (1/beta) else return (1/beta+n/beta*In_beta(n-1,beta))}
Cn_beta<-function(n,beta) {1/In_beta(n,beta)}
STn_beta<-function(x,n,beta){Cn_beta(n,beta)*(1+x)^n*exp(-x*beta)}
m<-10000
STn_beta_values<-0:m
T<-100
tvec<-T*0:m/m
for (i in 1:m) {
        STn_beta_values[i]<-STn_beta(i*T/m,n,beta)
}
freq<-10   # take out every 1000th point in the simulation & plot it
pick<-freq*(0:(m/freq))+1
title<-paste("ST(", round(n,1), ",",round(beta,1), ")")
plot(tvec[pick],STn_beta_values[pick],type='l',xlab="x",ylab="f(x)",main=title,ylim=c(0,0.2))
```

The graphics for the case $n=100$ and $\beta=2$ is given in Fig 1. The graphics for the case $n=40$ and $\beta=2$ is given in Fig 2.

5. Simulation of ST(n,β) processes.

To make simulation of $ST(n,\beta)$ process, the following algorithm can be applied:

1. Define interval $[0,T]$ of the simulation.
2. Set $k=0.$
3. While $\sum_{i=1}^{k}\tau_i < T$ do:

    3.1. Set $k=k+1$

    3.2. Generate $\tau_k \in ST(n-1,\beta).$

    3.3. Set $Y_k=1$ for standard $ST(n,\beta)$ process or simulate Y based on a given distribution f for compound $ST(n,\beta)$ process.

Then the trajectory of $X(t)\in ST(t;n,\beta)$ is given by the formula

$$X(t)=\sum_{i=1}^{N(t)}Y_i,$$

where

$$N(t)=\sum_{i}1_{\{T_i<T\}}.$$

Here is the code for creating graphic representation of the $ST(n,\beta)$ process:

```
#ST(n+1,beta)_process simulation
# Theta_grid_sets Poisson, Erlang, ST
T<-1000
lambda<-0.1
Theta_Poisson <-rexp(1,lambda)
while(max(Theta_Poisson)<T) Theta_Poisson <-c(Theta_Poisson,max(Theta_Poisson)+rexp(1,lambda))
m<-length(Theta_Poisson)
Theta_Poisson[m]<-T

# Erlang
beta_ER<-lambda
n_ER<-2
m_ER<-m/n_ER
Theta_ER<- Theta_Poisson[n_ER]
for (i in 2:m_ER)
{
Theta_ER<- c(Theta_ER, Theta_Poisson[i*n_ER])
}
```

```
#ST
ST_n<-5
ST_beta<-0.1
n<-20
tvec<-T*0:n/n
In_beta<-function(n,beta, recursive=TRUE)
{if (n==0) return (1/beta) else return ((1/beta)+(n/beta)*In_beta(n-1,beta))}
Cn_beta<-function(n,beta) {1/In_beta(n,beta)}
FDn_inverse<-function(y,n,beta){
Dn_values<-1:(n+1)
Dn_prob<-1:(n+1)
for (i in 1:(n+1))
{
Dn_prob[i]<-(Cn_beta(n,beta)*factorial(n)/(beta^i*factorial(n-i+1)))
}
Prob_acum<-Dn_prob[1]+Dn_prob[2]
for (k in 1:n)
{
if (y< Prob_acum) return (k)
else
Prob_acum<-Prob_acum+Dn_prob[k+2]
}
return(n+1)
}


#Dn_simulation<-function(n,ST_n,ST_beta){
Dn_values<-1:(n+1)
Dn_prob<-1:(n+1)
Dn_simulated<-1:n
for (i in 1:(n+1))
{
Dn_prob[i]<-(Cn_beta(n,ST_beta)*factorial(n)/(ST_beta^i*factorial(n-i+1)))
}
for (i in 1:n)
{
St_unif_value<- runif(1, min = 0, max = 1)
Dn_simulated[i]<-FDn_inverse(St_unif_value,ST_n,ST_beta)
}
acc_ST<- Dn_simulated[1]

Theta_ST<- Theta_Poisson[acc_ST]

while(acc_ST<m)
{
acc_ST<-(acc_ST+Dn_simulated[i])
if (acc_ST<=m)
Theta_ST<- c(Theta_ST, Theta_Poisson[acc_ST])
}
Theta_ST
s<-8000
tvec<-T*0:s/s
N<-0:s
N[1]<-0
u<-length(Theta_ST)

for (i in 1:s)
{
```

```
        N[i]<-0
        for (j in 1:u)
        {
                if ((T*(i-1)/s)> Theta_ST[j]) N[i]<-(N[i]+1)
        }
}
title<-paste("Траектория на ST процес n=",round((ST_n+1),1), ", beta=", round(ST_beta,2))
plot(tvec[1:(s-1)],N[1:(s-1)],type='l', xlab="t",ylab="N(t)", main=title)
```

The graphics of a trajectory of $ST(n,\beta)$ process for the case $n=6$ and $\beta=0.1$ is given in Fig 3.

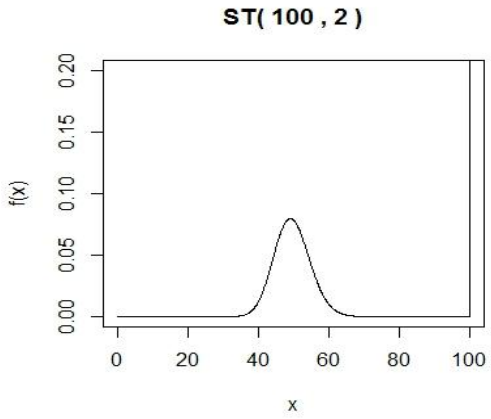


Figure 1. The graphics of $ST(n,\beta)$ distribution for the case $n=100$ and $\beta=2$.

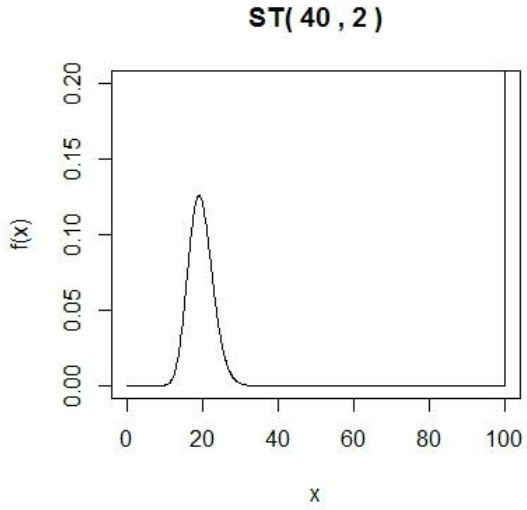


Figure 2. The graphics of $ST(n,\beta)$ distribution for the case $n=40$ and $\beta=2$.

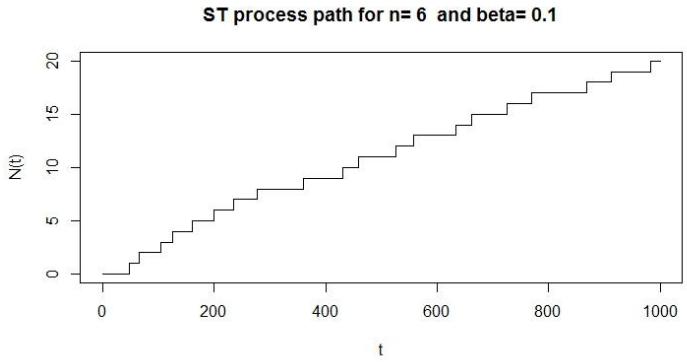


Figure 3. The graphics of a path of $ST(n,\beta)$ process for the case $n=6$ and $\beta=0.1$.

Conclusion

Switch time distributions presented in the article posses some suitable properties for modeling more complicated switch time structures in additive processes and their generalizations.

The present work may be extended for typical (not degenerated) ST processes as well as by studying other choices of G^k.

For example, we may choose $G^k \in NB(k,e^{-\beta})$. In this case, $\xi \mid D^n \equiv NB(D^n,e^{-\beta})$ and we obtain a distribution which we will call $ST2(n,\beta)$ distribution.

As another example we may consider $G^k \equiv \delta_k(x)$ - random variable which takes value k with probability one. Then $\xi \equiv D^n \in D^n(\beta)$. In this case, we say that random variable ξ has $ST3(n,\beta)$ distribution and denote $\xi \in ST3(n,\beta)$.

References

Grigelionis, B. On Non-Linear Filtering Theory and Absolute Continuity of Measures, Corresponding to Stochastic Processes. Proceedings of the Second Japan-USSR Symposium of Probability Theory, Lecture Notes in Mathematics 330, 80-94. Springer, Berlin. 1973.

Kallsen, J. Semimartingale Modelling in Finance. PhD thesis, Freiburg, Universität Freiburg. 1988.

Lévy, P. Théorie de l'Addition des Variables Aléatoires. Gauthier Villars. Paris. 1934.

Lévy, P. Processus Stochastiques et Mouvement Brownien. Edition Jacques Gabay: Sceaux. 1992.

Sato, K. Lévy processes and infinitely divisible distributions. Cambridge University Press: Cambridge , UK. 1999.

Stoynov, P. Negative binomial distributions and applications. Proceedings of Third International conference "Financial and Actuarial Mathematics – FAM – 2010" (18-20.09.2010, Sofia, Bulgaria) – Part One, PERUN-SPRINT PUBLISHING, Sofia, 2010, 12-20.

Stoynov, P. Mixed Negative Binomial distribution by weighted gamma mixing distribution. - Proceedings of the Fortieth Spring conference of the Union of Bulgarian Mathematicians – Borovets, Bulgaria, 5-9 April, 2011, 327-331.